Behavioral Economics

Víctor González-Jiménez[1]

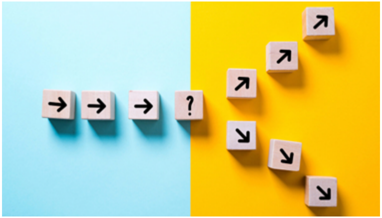
[1]Erasmus School of Economics, Erasmus University

Lecture 1: Reference Dependence

Today's Topics

1. Traditional Models: Expected Value and Expected Utility.
2. Behavioral Model: Reference Dependence.
   - Empirical Phenomena: Rabin's Paradox and Gains and Losses.
   - Components of the model: Loss Aversion and Reference Points.

Decisions under uncertainty

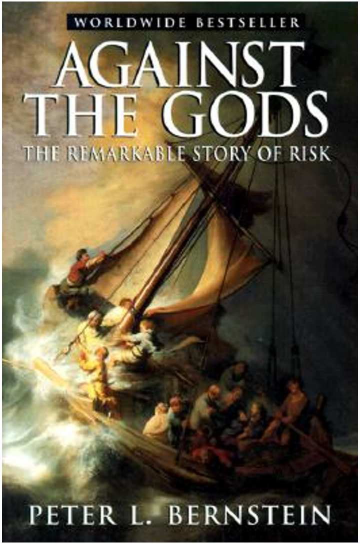


- Many economic decisions have uncertain consequences.
  - Purchasing insurance.
  - Buying stocks.
  - Buying a house.
  - Pursuing an education
- **How do individuals make decisions under uncertainty?**

Risk vs. Ambiguity

- We differentiate between two types of uncertainty.
  - **Risk:** known probabilities.
    - The probability of obtaining black (or red) in the game of roulette.
    - The probability of obtaining "3" in the throw of a six-sided dice.
  - **Ambiguity:** probabilities are subjective.
    - The probability of a calamity
    - The probability that Ajax wins the Eredivisie
- **We will first study decisions under risk.**

Risk



- Origins of risk management: Religious sacrifices.
- A nice book if you want to learn more about the origins of "risk."

A criterion for Decision under Risk

(a) Blaise Pascal

(b) Pierre de Fermat

(c) Cristiaan Huygens

A criterion for Decision under Risk

- **Problem of points**
  - Two players who contribute equally to a pot.
  - They have equal chance to win a game in each round.
  - They ex-ante agree that the one who first wins a certain number of rounds wins the pot.
  - An external event interrupts the game before there is a winner.
  - How does one divide the pot fairly?
- Pascal and Fermat discussed a solution in 1654.
- Huygens exposed a similar solution in 1657.
- The solution is **Expected Value**.

1.Traditional Models for Decision under Risk

- Expected Value
- Expected Utility

Expected Value

- Let $L = (p_1, x_1; ...; p_n, x_n)$ be a lottery yielding x_i with probability p_i where $i = 1, 2, ..., n$.
- Note that $p_1 + ... + p_n = 1$.
- The expected value of L is

$$\mathbb{E}(L) = p_1 \cdot x_1 + ... + p_n \cdot x_n$$

Expected Value in the Problem of Points

- The solution to the “problem of points” consists on calculating the probability of winning.
    - the stakes received by a player should be proportional to his/her probability of winning if the game were to continue at the point of stopping.

Expected Value in the Problem of Points (cont'd)

- Suppose that :
  - each player contributes 32 Euro.
  - the first player who wins four points take the entire stakes.
  - player A has 2 points.
  - player B has 1 point.
- The solution consists on calculating the probability that A wins 2 points before B wins 3 points.

Expected Value in the Problem of Points (cont'd)

- We only consider the next four rounds $(3+2-1)$.
- Let S be the event A winning a round and F the complement event. There are 16 possible outcomes.

1	SSSS	*
2	SSSF	*
3	SSFS	*
4	SSFF	*
5	SFSS	*
6	SFSF	*
7	SFFS	*
8	SFFF	
9	FSSS	*
10	FSSF	*
11	FSFS	*
12	FSFF	
13	FFSS	*
14	FFSF	
15	FFFS	
16	FFFF	

- Player A wins in eleven of the outcomes (the ones with asterisk). This gives $11/16 = 0.6875$.
- The expected value of A winning is $0.6875 \cdot 64 = 44$. Give A 44 Euros.

St. Petersburg Paradox

- Expected value was the accepted approach for decision under risk before Daniel Bernoulli proposed the following game:
  - A coin is tossed.
  - If "Heads" then you win 2 Euro if "Tails" you toss the coin again.
  - If "Heads" then you win 4 Euro if "Tails" you toss the coin again.
  - If "Heads" then you win 8 Euro if "Tails" you toss the coin again.
  - ...(ad infinitum)
- **How much would you pay to play this game?**

St. Petersburg Paradox (cont'd)

- The expected value of the game is:

$$\mathbb{E}(G) = \frac{1}{2} \cdot 2 + \frac{1}{4} \cdot 4 + \frac{1}{8} \cdot 8 + \frac{1}{16} \cdot 16 + ... = \infty$$

- The avg. price people are willing to pay is not higher than 25 euros.
  - On average your willingness to pay is $22'222,447$. When two individuals are taken out of the analysis it drops to 3.55 euros.
- The paradox is the discrepancy between what people seem willing to pay to enter the game and the infinite expected value.

Bernoulli's resolution

Instead of

$$\mathbb{E}(G) = \sum p_i x_i = \frac{1}{2}2 + \frac{1}{4}4 + \frac{1}{8}8 + ...$$

consider

$$\mathbb{E}\left(u(G)\right) = \sum p_i x_i = \frac{1}{2}u(2) + \frac{1}{4}u(4) + \frac{1}{8}u(8) + ...$$

Bernoulli's resolution (cont'd)

- The function u is inversely proportional to the amount of money
$$u'(x) = \frac{1}{x},$$

- or (up to constants),
$$u(x) = \ln(x).$$

So,
$$\mathbb{E}\left(u(x)\right) = \sum_i p_i ln(x_i) = \frac{1}{2} \cdot \ln(2) + \frac{1}{4} \cdot \ln(4) + \frac{1}{8} \cdot \ln(8) + ... < \infty$$

Expected Utility (EU)

(a) Daniel Bernoulli

(b) von Neumann and Morgenstern

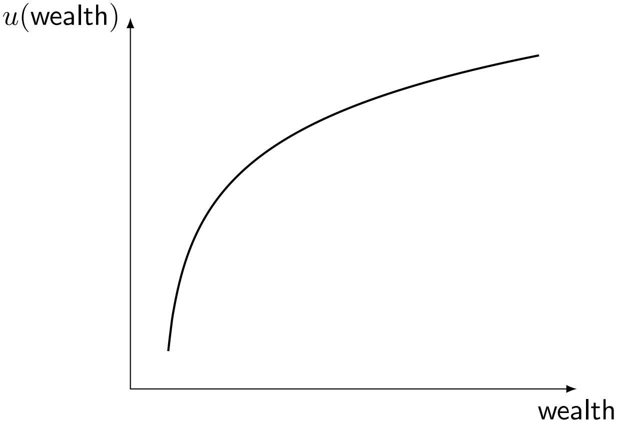
Expected Utility (EU)

- Agents act as if they multiply the utility of each possible outcome by its probability.
- Outcomes are expressed in terms of final wealth.
- Let $L := (p_1, x_1; ...; p_n, x_n)$ be a lottery yielding x_i with probability p_i where $i = 1, 2, ..., n$..
- Note that $p_1 + ... + p_n = 1$.
- W is the initial wealth and $W + x_i$ is the final wealth if the decision maker gets x_i
- The expected utility of L is

$$\mathbb{E}\left(u(L)\right) = p_1 \cdot u(W + x_1) + ... + p_n \cdot u(W + x_n)$$

Expected Utility

- Used in most economic theories and applications when risk is involved.
- Generally, u is assumed to be concave



Risk attitude under Expected Utility

- Consider an expected utility individual who receives the lottery $(0.5, 100; 0.5, 0)$.

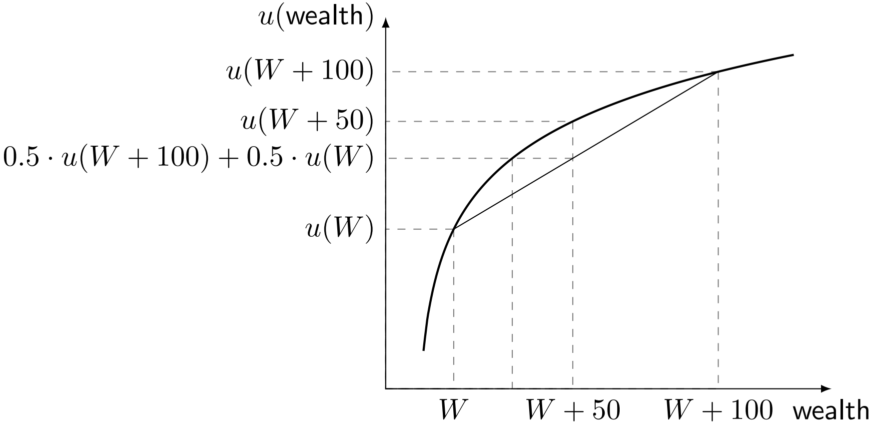


Figure: Risk attitudes under EU

Risk attitude and Expected Utility

- Risk attitudes:
  - **Risk aversion**: the individual prefers the expected value of a lottery to the lottery itself.
    - In other words, the individual prefers the expected value of $(p_1, x_1, ..., p_n, x_n)$ for sure rather than getting $(p_1, x_1, ..., p_n, x_n)$
    - e.g. prefers 50 Euro for sure rather than $(0.5, 100; 0.5, 0)$.
  - **Risk seeking**: the individual prefers a lottery to the expected value of that lottery.
  - **Risk neutrality**: the individual is indifferent between the expected value of a lottery and the lottery itself.

Risk attitude under Expected Utility

Under expected utility the following equivalences are true

- risk aversion $\Leftrightarrow$ u is concave.
  - In the previous figure u concave implied $0.5 \cdot u(W) + 0.5 \cdot u(W + 100) < u(W + 50)$
  - (the individual prefers the expected value of the lottery for sure to the expected utility of the lottery)
- risk seeking $\Leftrightarrow$ u is convex.
- risk neutral $\Leftrightarrow$ u is linear.

2. Behavioral Model: Reference Dependence

- Empirical Phenomena: Rabin's Paradox and Gains and Losses
- Components: Loss aversion and Reference Points

Two Empirical Phenomena

- **Phenomenon 1.** Rabin's Paradox
  - Violation of EU with u defined on final wealth and concave.
- **Phenomenon 2.** Gains and Losses
  - Violation of EU with u defined on final wealth.

Phenomenon 1

- You have initial wealth W. You reject the lottery $(0.5, 11; 0.5, -10)$.
  - (56% of you made that choice in the experiment)
- Under expected utility that choice implies

$$u(W) > 0.5 \cdot u(W + 11) + 0.5 \cdot u(W - 10),$$

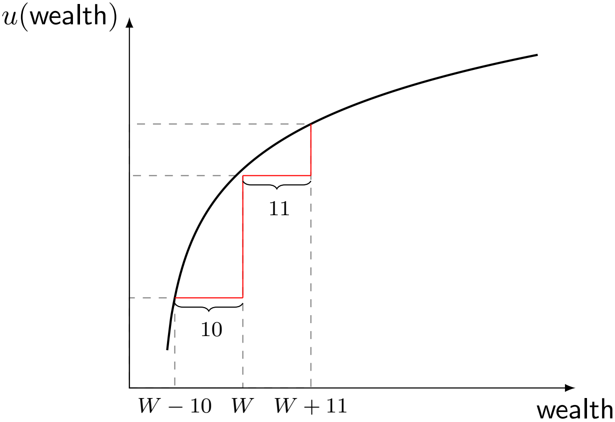
which can be written as

$$u(W) - u(W - 10) > u(W + 11) - u(W).$$

  - 11 Euros added to W add less utility than 10 Euros added to $W - 10$.
    - By concavity, marginal utility must be decreasing
  - On average, you value each Euro between W and $W + 11$ by at most $10/11$ as much as the Euros between $W - 10$ and W.
  - So, we can conclude that the extra utility of one Euro added to $W + 10$ is worth at most $10/11$ of the extra utility of one Euro added to $W - 10$.

Phenomenon 1 (cont'd)

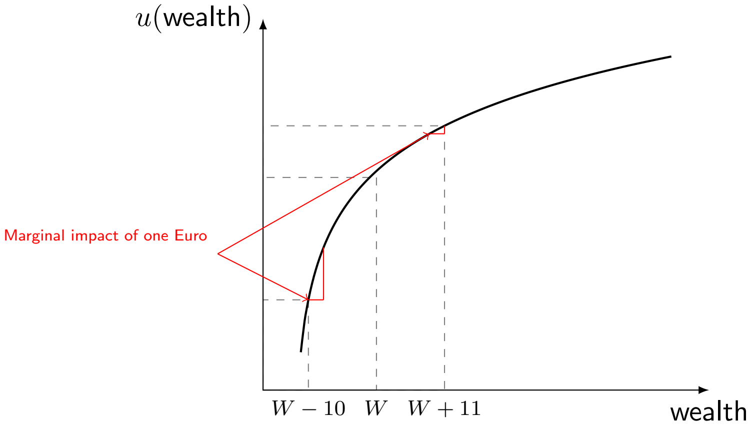
- On a graph:



- From this figure it is clear that 10 Euros added to $W - 10$ bring more utility than 11 Euros added to W.

Phenomenon 1 (cont'd)

- On a graph:



- This figure shows that the extra utility of one Euro added to $W + 10$ is worth at most $10/11$ of the extra utility of one Euro added to W–10.

Phenomenon (cont'd)

- Suppose you make the same choice when you are 21 Euros richer.
- Under EU

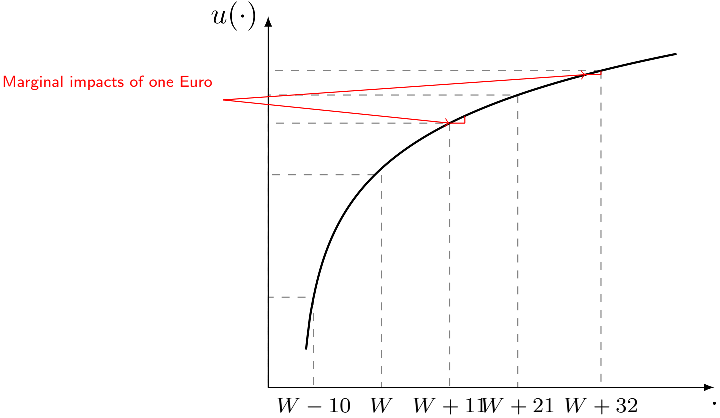
$$u(W + 21) > 0.5 \cdot u(W + 21 + 11) + 0.5 \cdot u(W + 21 - 10)$$

$$\Rightarrow u(W + 21) - u(W + 11) > u(W + 32) - u(W + 21)$$

  - On average, you value each Euro between $W + 32$ and $W + 21$ by at most $10/11$ as much as the Euros between $W + 11$ and $W + 21$.
    - Concavity implies that the extra utility of one Euro added to $W + 31$ is worth at most $10/11$ of the extra utility of one Euro added to $W + 11$.
  - We can conclude that the extra utility of one Euro added to $W + 31$ is worth at most $(10/11)^2 \approx 5/6$ of the extra utility of one Euro added to $W - 10$.

Phenomenon 1 (cont'd)

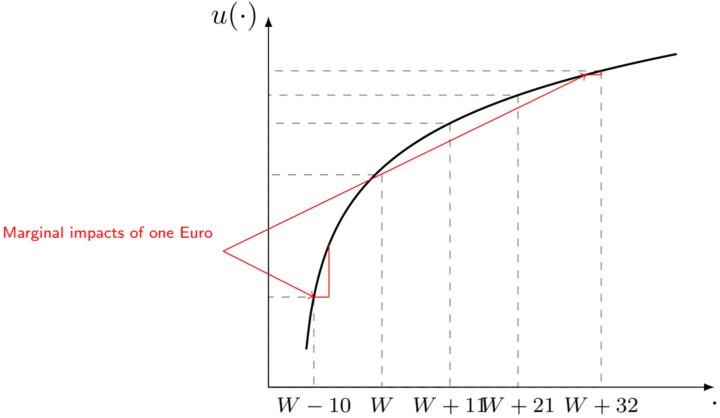
- On a graph:



- This figure shows that the extra utility of one Euro added to $W+31$ is worth at most $10/11$ of the extra utility of one Euro added to $W+11$.

Phenomenon 1 (cont'd)

- On a graph:



- This figure shows that the extra utility of one Euro added to $W+31$ is worth at most $5/6$ of the extra utility of one Euro added to $W-10$.

Phenomenon 1

- Iterating this forward gives absurd implications!
  - Marginal utility becomes quickly extremely small.
- Rejecting $(0.5, -10; 0.5, 11)$ implies a 9% decline in marginal utility for each 21 euro in additional initial wealth.
  - 42 euro richer: $(10/11)^2 \approx 5/6$
  - 420 euro richer: $(10/11)^{20} \approx 3/20$
  - 840 euro richer: $(10/11)^{80} \approx 2/100$
- You care less than 2% as about an additional euro when you are 900 euro richer than you are now.

Absurd implications from expected utility

Table 1

The Necessary, Implausible Consequences of Risk Aversion at Low Levels of Wealth

If an Expected Utility Maximizer Always Turns Down the 50/50 bet . . .	*Then She Always Turns Down the 50/50 Bet . . .*
lose \$10/gain \$10.10	lose \$1,000/gain \$∞
lose \$10/gain \$11	lose \$100/gain \$∞
lose \$100/gain \$101	lose \$10,000/gain \$∞
lose \$100/gain \$105	lose \$2,000/gain \$∞
lose \$100/gain \$110	lose \$1,000/gain \$∞
lose \$1,000/gain \$1,010	lose \$100,000/gain \$∞
lose \$1,000/gain \$1,050	lose \$20,000/gain \$∞
lose \$1,000/gain \$1,100	lose \$10,000/gain \$∞
lose \$1,000/gain \$1,250	lose \$6,000/gain \$∞
lose \$10,000/gain \$11,000	lose \$100,000/gain \$∞
lose \$10,000/gain \$12,500	lose \$60,000/gain \$∞

Phenomenon 1- Conclusion

- Consequences of those choices:
  - Expected utility captures risk through the concavity of u.
  - This leads to unreasonable predictions.
  - So, EU with u being defined over wealth and being concave is not the right way to describe peoples' behavior.

Phenomenon 2

- **Question 1** Would you accept the following lottery: $(0.5, 15; 0.5, -10)$?
  - 37% of you accepted the lottery in the experiment.
  - The typical experimental finding is that people reject the lottery.
- **Question 2** Would you accept the lottery in Question 1 if your wealth was lower by 10 Euro?
  - 37% of you accepted the lottery in the experiment.
  - The typical experimental finding is that people reject the lottery.

Phenomenon 2 (cont'd)

- **Question 3** Which lottery would you rather have: $L = (0.5, 5; 0.5 - 20)$, or $R = -10$?
  - 86% of you chose L in the experiment.
  - The typical experimental finding is that people choose L.
- **Question 4** Would you play lottery L or R if your wealth was higher by 10 Euro?
  - 67% of you chose L in the experiment.
  - The typical experimental finding is that people choose L.

Phenomenon 2 (cont'd)

- Those choices lead to contradictions under EU:
  - **Question 1**: $u(W) > 0.5 \cdot u(W + 15) + 0.5 \cdot u(W - 10)$.
  - **Question 2**: $u(W - 10) > 0.5 \cdot u(W + 5) + 0.5 \cdot u(W - 20)$.

  - **Question 3**: $u(W - 10) < 0.5 \cdot u(W + 5) + 0.5 \cdot u(W - 20)$.
  - **Question 4**: $u(W) < 0.5 \cdot u(W + 15) + 0.5 \cdot u(W - 10)$.

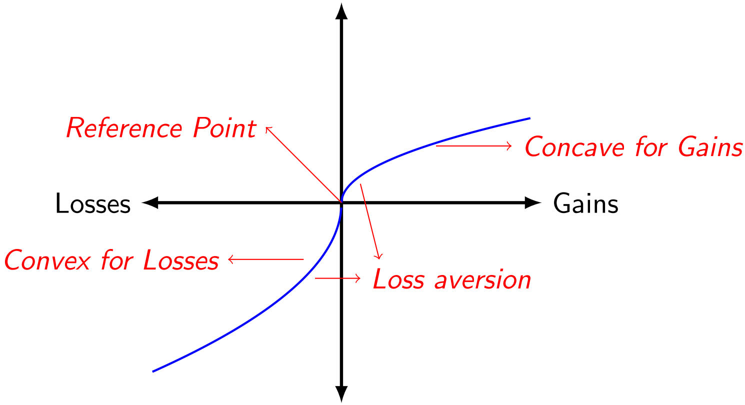
Phenomenon 2 (cont'd)

- Therefore, most people do not think in terms of final wealth but consider **changes of wealth (Gains and Losses).**
- Note that less people are willing to take a risk when one option involves no losses.

Kahneman and Tversky's explanation

- Expected Utility cannot accommodate Phenomenon 2 if utility is defined on final wealth.
- Kahneman and Tversky (1979) argued that:
  - u should be defined in terms of changes of wealth with respect to a **Reference Point** (RP).
  - People's behavior differ between gains and losses $\Rightarrow$ Loss aversion.

“S-shaped” utility function



Components “S-shaped” utility function.

- **Reference point**
  - Point of comparison
- **Loss aversion**
  - Utility is steeper for losses than for gains
  - losses loom larger than commensurate gains in the decision-maker’s utility.
- **Diminishing sensitivity** (for outcomes)
  - Utility is concave in gains and convex in losses...
  - ...generates risk averse attitudes in gains and risk seeking in losses.

Resolution Phenomenon 1.

- Assume:

$$u(x) = \begin{cases} \sqrt{x} & \text{if gains,} \\ -\lambda \cdot \sqrt{-x} & \text{if losses.} \end{cases}$$

  - Let $RP = W$. Rejecting $(0.5, 11; 0.5, -10)$ implies

$$\sqrt{0} > 0.5\sqrt{11} - \lambda 0.5\sqrt{10}$$

  - This decision can be fully explained by λ, the loss aversion parameter;
    - it must exhibit $\lambda \geq \sqrt{11}/\sqrt{10}$
  - The same inequality holds when you are 21 Euro richer if $RP = W + 21$ without having to change the magnitude of λ.
  - The same inequality holds when you are 42 Euro richer if $RP = W + 42$ without having to change the magnitude of λ.
  - ...
- No contradictions :)

Resolution Phenomenon 2.

- Assume:

$$u(x) = \begin{cases} \sqrt{x} & \text{if gains,} \\ -2 \cdot \sqrt{-x} & \text{if losses.} \end{cases}$$

  - For **Question 1** let $RP = W$, then
  $$\sqrt{0} > 0.5 \cdot \sqrt{15} - 0.5 \cdot 2 \cdot \sqrt{10} = -1.226$$
  - For **Question 2** let $RP = W - 10$, then
  $$\sqrt{0} > 0.5 \cdot \sqrt{15} - 0.5 \cdot 2 \cdot \sqrt{10}$$
  - For **Question 3** let $RP = W$, then
  $$-2 \cdot \sqrt{10} = -6.324 < 0.5 \cdot \sqrt{5} - 0.5 \cdot 2 \cdot \sqrt{20} = -2.536$$
  - For **Question 4** let $RP = W + 10$, then
  $$-2 \cdot \sqrt{10} < 0.5 \cdot \sqrt{5} - 0.5 \cdot 2 \cdot \sqrt{20}$$
- No contradictions :)

Reference Points

- What is the reference point?
  - In many papers, reference point is the status quo or initial wealth.
- The proposal of Kőszegi & Rabin (2006) is different.
  - Reference point is what you expect ("recent expectations")
  - They propose a model without probability weighting (to be discussed in the next lecture) but with two parts
    - traditional consumption utility
    - gain-loss utility
  - This model is different than prospect theory
  - The reference point can be a random variable, or a lottery...

Reference Points and Effort Provision

- Abeler, Johannes; Falk, Armin; Goette, Lorenz; Huffman, David (2011) "Reference Points and Effort Provision," American Economic Review 101, 470–492.

Reference Points and Effort Provision

- In an experiment they test Kőszegi & Rabin's (2006) idea.
  - Participants know that they will be paid a wage w times their effort e (with probability $1/2$) or a fix amount f.
  - Cost of working is $c(e)$
  - Traditional model with linear utilities:
    $U = 1/2 \cdot \big(we + f\big) - c(e)$.
    - First order condition: $w/2 – c'(e) = 0$
- They proposed, based on Kőszegi & Rabin's (2006), that the reference point is the lottery $(0.5, we; 0.5, f)$:

$$U = \begin{cases} \frac{(we+f)}{2} - c(e) + \frac{(we-f)-\lambda\cdot(we-f)}{4} & \text{if } we \geq f \\ \frac{(we+f)}{2} - c(e) + \frac{(f-we)-\lambda\cdot(f-we)}{4} & \text{if } we < f. \end{cases}$$

  - **Prediction**: when $we < f$ marginal returns to effort are higher than $w/2$ (the return to effort in the traditional model)
  - Confirmed in an experiment.

Reference Points

- What is the reference point?
- Baillon et al. (2020) investigate this question in an experiment.
  - Different candidates: status quo (initial wealth), expected value, Kőszegi & Rabin's (2006), max-min, min-max, max probability.
- Two candidates stand out:
  - Status quo or initial wealth (40% of subjects) and Max-min (31% of subjects),

The End!

Today's recommendation

- Snarky Puppy's "We like it here" (2013).
- **Interesting economics fact:** The band created their own record label (GROUNDUP) to help less-known artists capitalize on their growing fanbase.