

# Behavioral Economics

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## Lecture 2: Probability Weighting Functions

# Today's Topics

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- Empirical phenomena: Allais' aradox and fourfold pattern of risk
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  - Empirical phenomena: Allais' aradox and fourfold pattern of risk
  - Probability weighting function.
2. Prospect Theory
3. Applications

# Risk

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- We continue studying choice under risk.
- We now discuss further deviations from Expected Utility.
  - Expected Utility is the traditional model of choice under risk.
  - The behaviors we will discuss today cannot be explained by reference dependence (S-shaped function).

# 1. Behavioral Model: Probability Weighting Function

- Empirical Phenomena: Allais' paradox and Fourfold pattern of risk
- Probability Weighting Function

# Phenomenon 3

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- **Problem 1** which lottery would you choose?
  - Option *A*. 100% chance to win 10 Euro
  - Option *B*. 10% chance to win 50 Euro, 89% chance to win 10 Euro and 1% chance to win 0 Euros.

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- Only 7% of you chose that option :( .

# Phenomenon 3

- **Problem 2** which lottery would you choose?
  - Option *A*. 11% chance to win 10 Euro, and 0 Euro otherwise.
  - Option *B*. 10% chance to win 50 Euro, and 0 Euro otherwise.
- In this problem, option *B* is typically preferred to *A*.

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- **Problem 2** which lottery would you choose?
  - Option *A*. 11% chance to win 10 Euro, and 0 Euro otherwise.
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- 100 % of you chose that option.

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- In **Problem 2**, Option  $B$  is typically preferred to Option  $A$ .
  - This preference implies

$$0.11 \cdot (W + 10) + 0.89 \cdot u(W) < 0.1 \cdot u(W + 50) + 0.9 \cdot u(W)$$

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- Paradox! proposed by Maurice Allais (1953).

## Phenomenon 3: Allais Paradox

- The S-shaped utility that we discussed in the previous lecture cannot solve this paradox.
- If you assume  $RP = W$ , the same contradiction will emerge.
  - in the equations of the previous slide  $W$  is dropped everywhere and the inequalities hold.

# Phenomenon 4

- **Problem 3.** which of the lotteries would you choose?
  - Option *A*. 100% chance to win 2 Euro
  - Option *B*. 10% chance to win 20 Euro and nothing otherwise.

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- **Problem 3.** which of the lotteries would you choose?
  - Option *A*. 100% chance to win 2 Euro
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- In this problem, option *B* is typically preferred to *A*.
- 57% of you chose that option :) .

# Phenomenon 4

- **Problem 4.** which lottery would you choose?
  - Option *A*. 100% chance to win 18 Euro
  - Option *B*. 90% chance to win 20 Euro and nothing otherwise.

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- **Problem 4.** which lottery would you choose?
  - Option *A*. 100% chance to win 18 Euro
  - Option *B*. 90% chance to win 20 Euro and nothing otherwise.
- In this problem, option *A* is typically preferred to *B*.
- 88% of you chose that option :) .

## Phenomenon 4

- **Problem 5** which lottery would you choose?
  - Option *A*. 100% chance to lose 2 Euro
  - Option *B*. 10% chance to lose 20 Euro and nothing otherwise.

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## Phenomenon 4

- **Problem 5** which lottery would you choose?
  - Option *A*. 100% chance to lose 2 Euro
  - Option *B*. 10% chance to lose 20 Euro and nothing otherwise.
- In this problem, option *A* is typically preferred to *B*.
- 51% of you chose that option.

## Phenomenon 4

- **Problem 6** which lottery would you choose?
  - Option *A*. 100% chance to lose 18 Euro
  - Option *B*. 90% chance to lose 20 Euro and nothing otherwise.

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## Phenomenon 4

- **Problem 6** which lottery would you choose?
  - Option *A*. 100% chance to lose 18 Euro
  - Option *B*. 90% chance to lose 20 Euro and nothing otherwise.
- In this problem, option *B* is typically preferred to *A*.
- 81% of you chose that option.

## Phenomenon 4: Fourfold Pattern

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- **Definitions (Review)**

- Risk averse: Prefer the expected value of a lottery to the lottery itself.
- Risk seeking: Prefer a lottery to the expected value of the lottery

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  - Risk averse: Prefer the expected value of a lottery to the lottery itself.
  - Risk seeking: Prefer a lottery to the expected value of the lottery
- In **Problem 3**, most people choose  $B$ .
  - Risk seeking
- in **Problem 4**, most people choose  $A$ .
  - Risk averse

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- Risk averse: Prefer the expected value of a lottery to the lottery itself.
  - Risk seeking: Prefer a lottery to the expected value of the lottery
- In **Problem 3**, most people choose  $B$ .
    - Risk seeking
  - in **Problem 4**, most people choose  $A$ .
    - Risk averse
  - In **Problem 5**, most people choose  $A$ .
    - Risk averse

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- In **Problem 3**, most people choose  $B$ .
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  - in **Problem 4**, most people choose  $A$ .
    - Risk averse
  - In **Problem 5**, most people choose  $A$ .
    - Risk averse
  - In **Problem 6**, most people choose  $B$ .
    - Risk seeking

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## Phenomenon 4: Fourfold Pattern

- The S-shaped utility function in the previous lecture is not enough to accommodate the fourfold pattern.

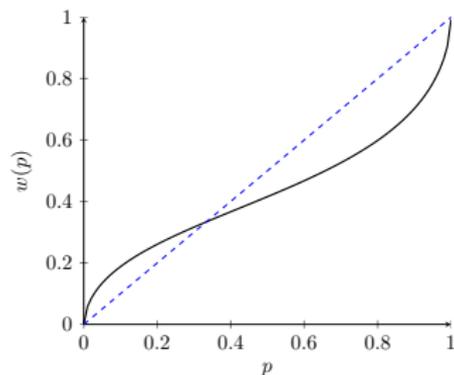
## Phenomenon 4: Fourfold Pattern

- The S-shaped utility function in the previous lecture is not enough to accommodate the fourfold pattern.
- Not only gains and losses matter but probabilities too!

# Probability Weighting Function

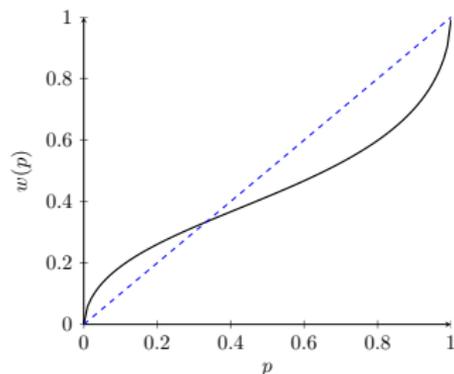
# Probability Weighting Function

- Outcomes are transformed by a utility function. Why not probabilities?



**Figure:** An example of a probability weighting function

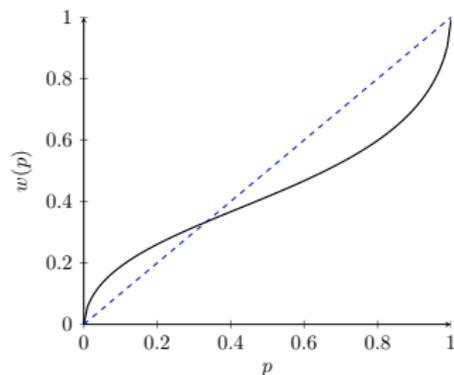
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**Figure:** An example of a probability weighting function

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  - A change from 100% to 99% may have a huge **psychological** impact: no certainty anymore. (See our Phenomenon 3).

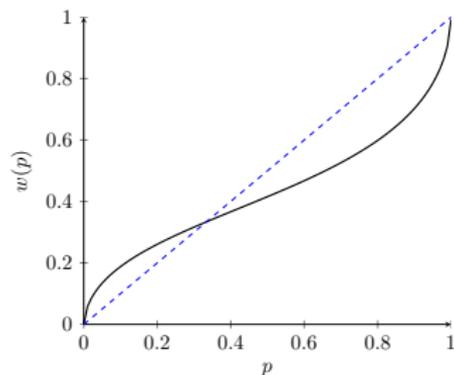
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**Figure:** An example of a probability weighting function

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  - Same from 0% to 1% (See the phenomenon 4).

# Probability Weighting Function



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  - A change from 100% to 99% may have a huge **psychological** impact: no certainty anymore. (See our Phenomenon 3).
  - Same from 0% to 1% (See the phenomenon 4).
  - But what about a change from 51% to 52%. Does it feel very different?

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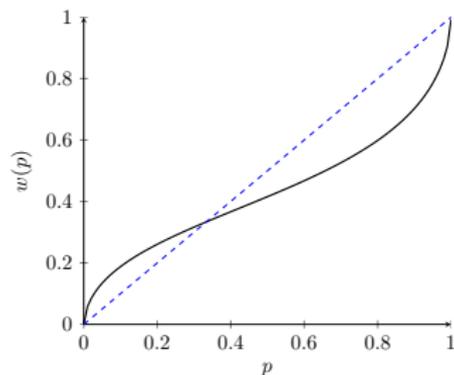


Figure: An example of a probability weighting function

- Outcomes are transformed by a utility function. Why not probabilities?
  - A change from 100% to 99% may have a huge **psychological** impact: no certainty anymore. (See our Phenomenon 3).
  - Same from 0% to 1% (See the phenomenon 4).
  - But what about a change from 51% to 52%. Does it feel very different?
- The probability weighting function should display that *diminishing sensitivity* to probabilities.

## 2. Prospect Theory

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  - A utility defined on gains and losses and not on final wealth, and with loss aversion.

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- We can now define Prospect Theory with its two components.
  - A utility defined on gains and losses and not on final wealth, and with loss aversion.
  - Probability weighting function.
    - Kahneman and Tversky (1979)'s version of prospect theory has some problems.
    - **We will focus on the 1992 version, also called Cumulative Prospect Theory.**



Figure: Amos Tversky & Daniel Kahneman (Nobel Prize 2002)

## Prospect Theory: 2 positive outcomes

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with  $u$  and  $w^+$  nondecreasing,  $u(0) = 0$ ,  $w^+(0) = 0$  and  $w^+(1) = 1$ .

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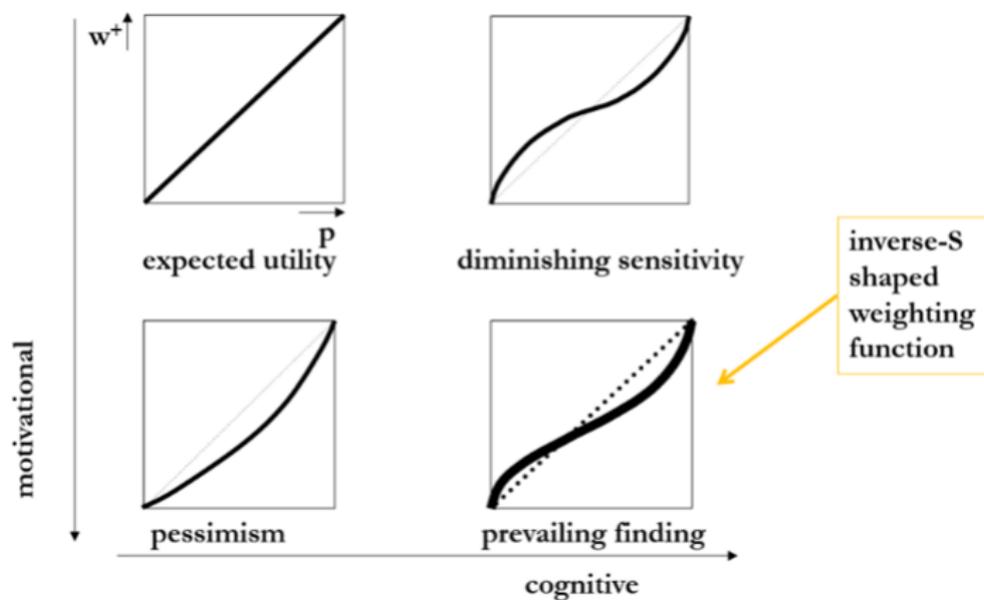
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- $w^+$  is the weighting function for gains.
- If  $w^+(p) = p$ , Prospect Theory becomes Expected Utility with  $u$  defined on changes of wealth.

# Typical weighting function for gains



## Prospect Theory: 2 negative outcomes

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- $w^-$  is the weighting function for losses.

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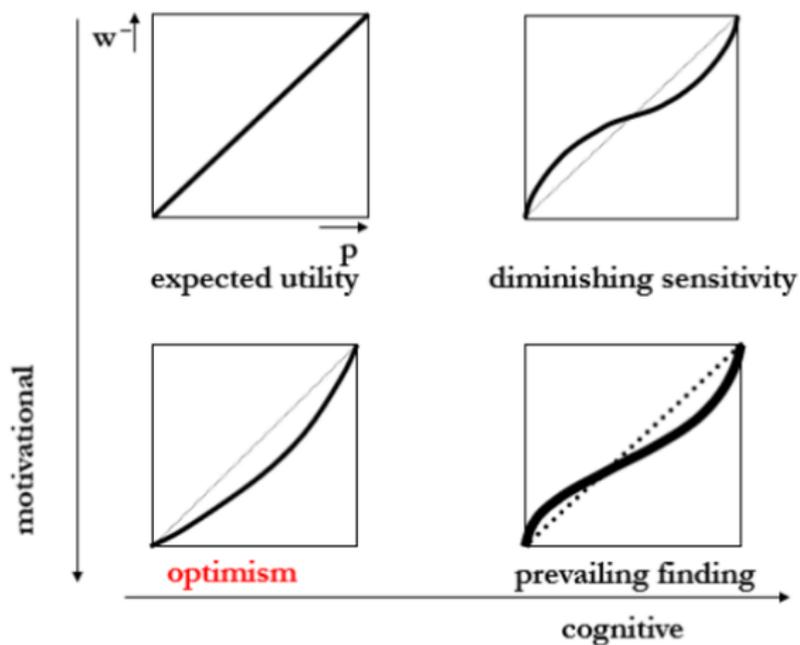
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- $w^-$  is the weighting function for losses.
- If  $w^-(p) = p$ , Prospect Theory becomes Expected Utility with  $u$  defined on changes of wealth.

# Typical weighting function for losses



## Prospect Theory: 3 positive outcomes

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with  $u$  and  $w^+$  like before.

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with  $u$  and  $w^+$  like before.

- Note that  $w^+(p_1 + p_2 + p_3) - w^+(p_1 + p_2) = 1 - w^+(p_1 + p_2)$ .

# Allais Paradox and Prospect Theory

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$$\begin{aligned}u(10) &> w^+(0.10) \cdot u(50) + (w^+(0.99) - w^+(0.10)) \cdot u(10) \\ &\quad + (1 - w^+(0.99)) \cdot u(0) \\ \Rightarrow (1 - w^+(0.99)) + w^+(0.10) &> w^+(0.10)u(50)\end{aligned}$$

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- In **Problem 2**,  $B$  is typically preferred to  $A$ 
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$$\Rightarrow w^+(0.11) \cdot u(10) < w^+(0.10)u(50)$$

- No paradox anymore, we just learned something about  $w^+$ .  
Namely, that

$$w^+(0.11) - w^+(0.10) < w^+(1) - w^+(0.99).$$

## Prospect Theory: 2 positive and 2 negative outcomes

- consider  $(p_1, x_1; \dots; p_4, x_4)$  with  $x_1 \geq x_2 \geq 0 \geq x_3 \geq x_4$

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- with  $u$ ,  $w^+$ , and  $w^-$  like before.

# Prospect Theory: General

- consider  $(p_1, x_1; \dots; p_n, x_n)$  such that
$$x_1 \geq x_2 \geq \dots \geq x_k \geq 0 \geq x_{k+1} \geq \dots \geq x_n$$
- Evaluated as

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 $x_1 \geq x_2 \geq \dots \geq x_k \geq 0 \geq x_{k+1} \geq \dots \geq x_n$
- Evaluated as

$$PT = \sum_{j=1}^k \left( w^+ \left( \sum_{i=1}^j p_i \right) - w^+ \left( \sum_{i=1}^{j-1} p_i \right) \right) \cdot u(x_j) + \\ \sum_{j=k+1}^n \left( w^- \left( \sum_{i=j}^n p_i \right) - w^- \left( \sum_{i=j+1}^n p_i \right) \right) \cdot u(x_j)$$

- with  $u$ ,  $w^+$ , and  $w^-$  like before.

# Prospect Theory: General

- For the exam: Remember

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  - The RP and the S-shaped utility.

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# Prospect Theory: General

- For the exam: Remember
  - The RP and the S-shaped utility.
  - The focus on extreme outcomes (both negative and positive) and the inverse-S shaped weighting function.
  - The way PT works based on the previous slides.

### 3. Applications of Prospect Theory

# Reference-dependence and marathon runners (Allen et al., 2016)

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## Reference-dependence and marathon runners (Allen et al., 2016)

- Marathon running is ideal to look for field evidence of reference dependence.
- Clear and stable reference points.
  - Runners think about their performance relative to round numbers.
  - A runner may feel significantly better about herself if she runs a 3:59 marathon as opposed to a 4:01 marathon



Figure: Run!

## Reference-dependence and marathon runners (Allen et al., 2016)

- 9,662,855 marathon finishes were analyzed.
- Chip technology to monitor times.

# Reference-dependence and marathon runners (Allen et al., 2016)

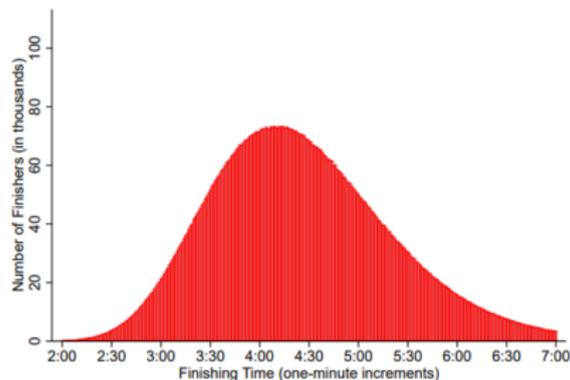
- 9,662,855 marathon finishes were analyzed.
- Chip technology to monitor times.
- 6888 marathons
  - from 1970-2013 (90.14% is 2000 or later).

	Total Marathon Sample		
	Mean	Std. Dev.	Observations
<b>Finishing time (HH:MM:SS)</b>	4:26:34	0:59:19	9,662,845
<b>Marathon year</b>	2005.98	6.46	9,662,845
<b>Age</b>	39.38	11.6	5,330,457
<b>Male (1 = Male, 0 = Female)</b>	0.66	0.48	7,968,042
<b>Split 10 kilometers (HH:MM:SS)</b>	1:02:23	0:17:57	2,068,431
<b>Split half marathon (HH:MM:SS)</b>	2:09:20	0:28:19	3,258,155
<b>Split 30 kilometers (HH:MM:SS)</b>	3:12:27	0:44:53	1,504,567
<b>Split 40 kilometers (HH:MM:SS)</b>	4:25:04	1:01:09	1,040,151

Figure: Descriptive statistics

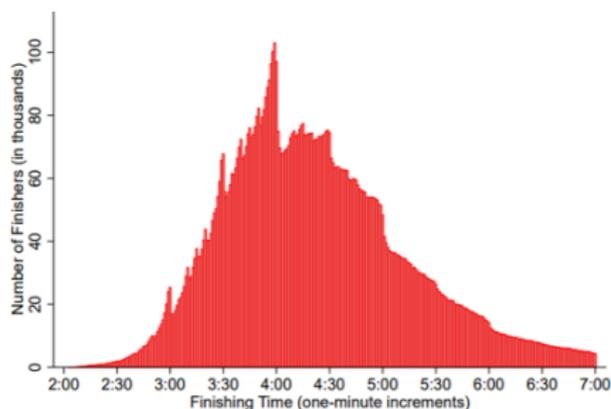
# Reference-dependence and marathon runners (Allen et al., 2016)

- Many marathons!
- Law of Large Numbers: would expect finishing times to resemble a smooth distribution



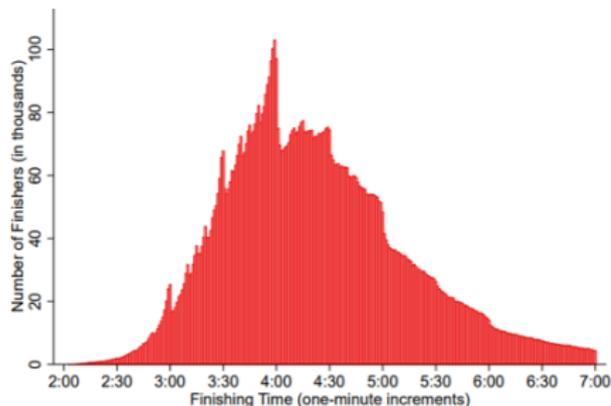
# Reference-dependence and marathon runners (Allen et al., 2016)

- Actual distribution looks like this.
  - Bunching below round (half-hour) finishing times.
- Consistent with reference dependence where reference point is a goal



## Reference-dependence and marathon runners (Allen et al., 2016)

- Focus on round numbers
  - easier to follow, unlike, say, the exact finishing time of a close friend
  - round numbers are frequently mentioned as goals by marathon runners themselves
- Clear excess mass just to the left of the 30-minute marks.



# Reference-dependence and marathon runners (Allen et al., 2016)

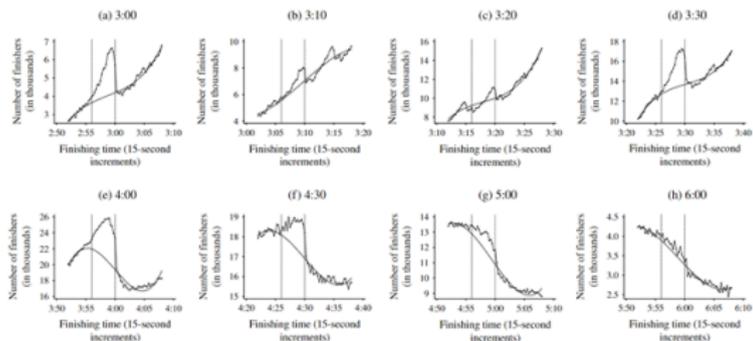


Figure: Bunching

- In the absence of reference-dependence, we would not observe this bunching in the data.
- In not so popular rounding times, the bunching is less severe.

# Reference-dependence and marathon runners (Allen et al., 2016)

- About 30% of runners increase their speed in the last 2.195 kilometers.
- This fraction of runners increases to almost 40% if a runner was right on target to finish at a round number.

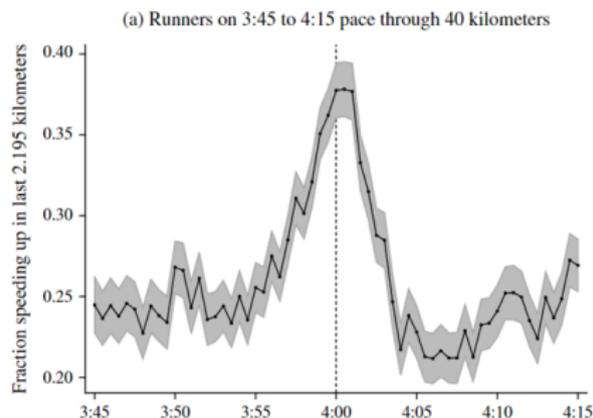


Figure: Speeding up

## Reference-dependence and marathon runners (Allen et al., 2016)

- runners who were just on pace to reach a round number were significantly less likely to slow down in the last leg of the marathon.

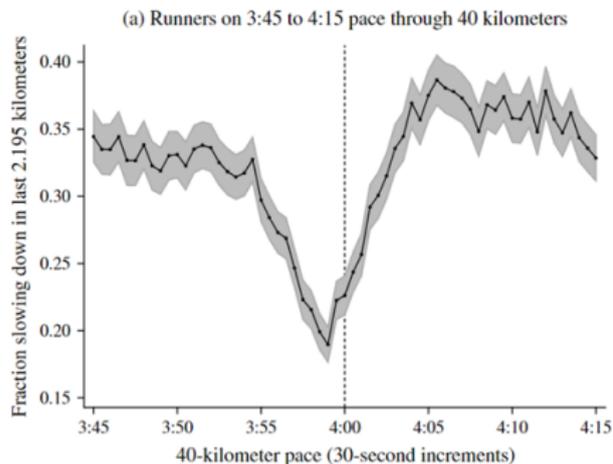


Figure: Slowing down

# Pay, Reference Points, and Police Performance (Mas, 2006)

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- Police unions and municipalities negotiate wage raises.
- In the eventuality of an impasse, both parties turn up to a final offer arbitration (FOA).
  - both parties submit offers to an arbitrator.
  - arbitrator chooses one of the offers.
- If the union's offer is the policeperson reference point, the arbitrator's decision can be regarded as a gain or a loss.
  - a loss can be demotivating (it generates strong disutility).

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- Data from New Jersey, USA.
  - 1978-1996
  - FOA between municipalities and unions.
  - data on police effectiveness: crimes cleared and felony arrests.

# Pay, Reference Points, and Police Performance (Mas, 2006)



- Prior to arbitration, Union and Employer municipalities had similar monthly clearance rates.

Figure: Comparison of Union and Employer City Average Clearance Rates

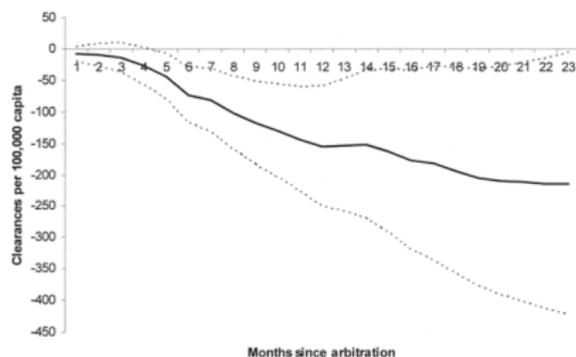
# Pay, Reference Points, and Police Performance (Mas, 2006)



- Prior to arbitration, Union and Employer municipalities had similar monthly clearance rates.
- After arbitration, Union municipalities clear more crimes.

Figure: Comparison of Union and Employer City Average Clearance Rates

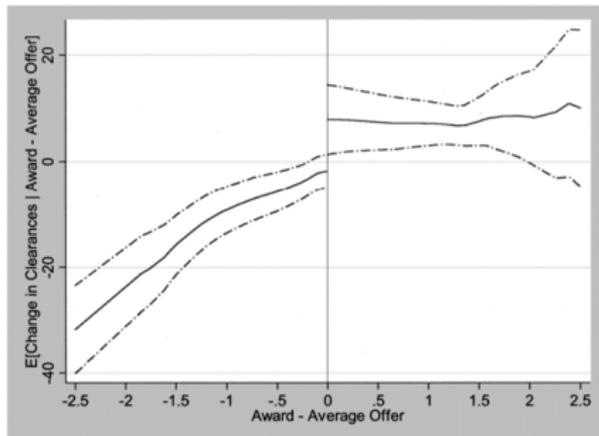
# Pay, Reference Points, and Police Performance (Mas, 2006)



- Difference in clearances between “Employer” and “Union” municipalities is more than 225 crimes cleared by arrest per 100,000 capita.

Figure: Difference in Clearances between Employer and Union Cities in Postarbitration Month

# Pay, Reference Points, and Police Performance (Mas, 2006)



- When police lose in arbitration, there is a positive relationship between effort and the gap between the pay raise demanded and the actual award.

Figure: Change in Clearances Conditional on the Deviation of the Award from the Average of the Offers

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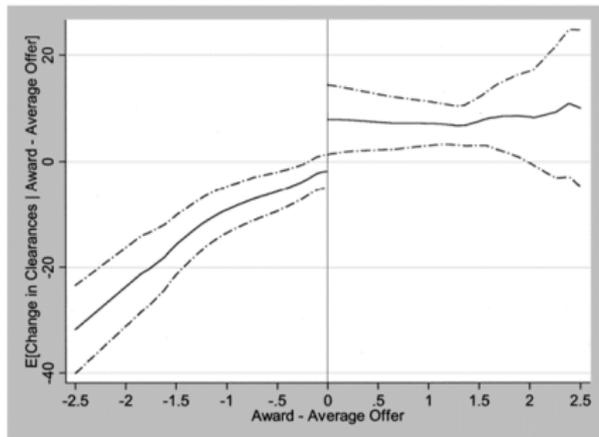


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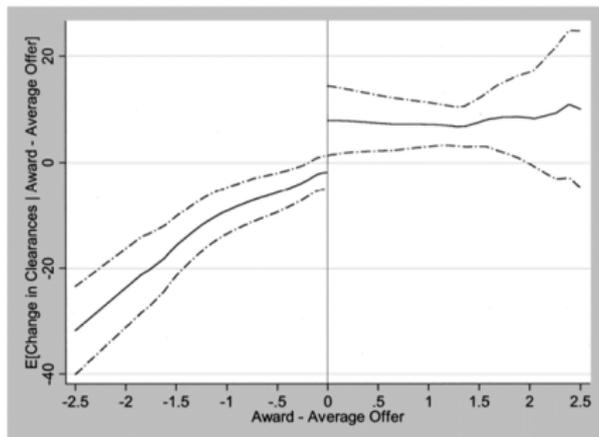
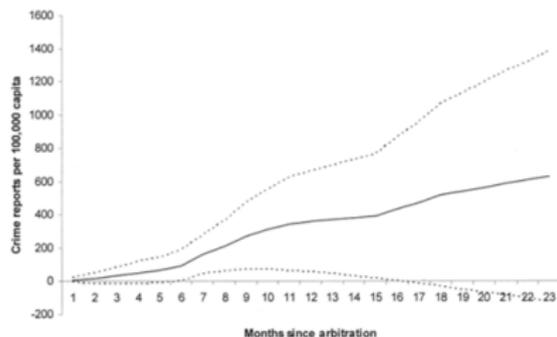


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- When police lose in arbitration, there is a positive relationship between effort and the gap between the pay raise demanded and the actual award.
- This is not the case when the award increment is higher than the police offer.
- Evidence that losses resonate more than gains.

# Pay, Reference Points, and Police Performance (Mas, 2006)



- There were more than 600 excess crime reports per 100,000 capita in Employer cities in the 23 months after arbitration

Figure: Cumulative Effect of Union Losses on Crime

# The Nature of Risk Preferences: Evidence from Insurance Choices (Barseghyan et al., 2013)

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- What preferences do consumers exhibit when facing downside risk situations?
- Are households' deductible choices influenced by probability weighting?

# The Nature of Risk Preferences: Evidence from Insurance Choices (Barseghyan et al., 2013)

- Data from large US insurance company.
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  - Information about:
    - Household
    - Claims
    - Policies

# The Nature of Risk Preferences: Evidence from Insurance Choices (Barseghyan et al., 2013)

- Deductible choices: Most households choose \$500.

Deductible	Auto collision	Auto comprehensive	Home
\$50		5.2	
\$100	1.0	4.1	0.9
\$200	13.4	33.5	
\$250	11.2	10.6	29.7
\$500	67.7	43.0	51.9
\$1,000	6.7	3.6	15.9
\$2,500			1.2
\$5,000			0.4

*Notes:* Values are percent of households. Core sample of 4,170 households.

Figure: Deductible choices

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- Deductible choices: Most households choose \$500.
- Probabilities: estimated from claims (next slide).
- Premiums vary across households  $\Rightarrow$  company determines price based on regulations and household ratings.

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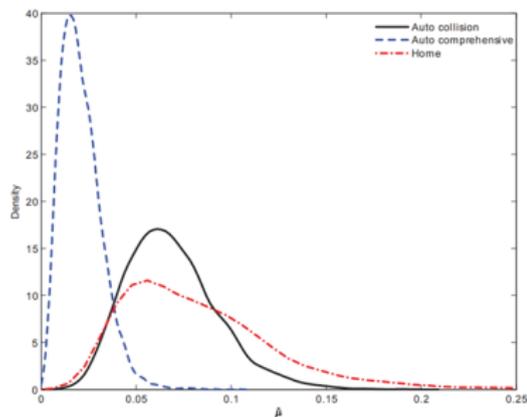


Figure: Estimated probabilities

	Mean	SD	1st percentile	99th percentile
Auto collision premium for \$500 deductible	180	100	50	555
Auto comprehensive premium for \$500 deductible	115	81	26	403
Home all perils premium for \$500 deductible	679	519	216	2,511
<i>Cost of decreasing deductible from \$500 to \$250:</i>				
Auto collision	54	31	14	169
Auto comprehensive	30	22	6	107
Home all perils	56	43	11	220
<i>Savings from increasing deductible from \$500 to \$1,000:</i>				
Auto collision	41	23	11	127
Auto comprehensive	23	16	5	80
Home all perils	74	58	15	294

Note: Annual amounts in dollars. Core sample of 4,170 households.

Figure: Premium Menus

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- Under Prospect Theory, households' preference over deductible choices are:

$$PT = (1 - \Omega^-(\mu))u(-p_d) + \Omega^-(\mu)u(-p_d - d)$$

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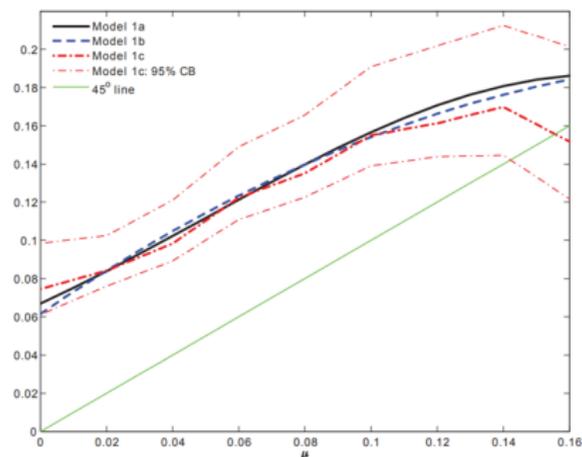
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# The Nature of Risk Preferences: Evidence from Insurance Choices (Barseghyan et al., 2013)

- Analysis with homogeneous preferences:
  - $\Omega^-(\mu)$  are the same for all households
  - Probability overweighting for small probabilities of calamities.



# The Nature of Risk Preferences: Evidence from Insurance Choices (Barseghyan et al., 2013)

	(1)	(2)	(3)	(4)	(5)
Standard risk aversion	$r = 0$	$r = 0.00064$	$r = 0$	$r = 0.00064$	$r = 0.0129$
Probability distortions?	No	No	Yes	Yes	No
$\mu$	WTP	WTP	WTP	WTP	WTP
0.020	10.00	14.12	41.73	57.20	33.76
0.050	25.00	34.80	55.60	75.28	75.49
0.075	37.50	51.60	67.30	90.19	104.86
0.100	50.00	68.03	77.95	103.51	130.76
0.125	62.50	84.11	86.41	113.92	154.00

*Notes:* WTP denotes—for a household with claim rate  $\mu$ , the utility function in equation (2), and the specified utility parameters—the household's maximum willingness to pay to reduce its deductible from \$1,000 to \$500 when the premium for coverage with a \$1,000 deductible is \$200. Columns 3 and 4 use the probability distortion estimates from Model 1a.

Figure: Economic Significance

Columns 3 and 4 reveal that probability weighting has a large economic impact.

# Exam-like question

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- Last Tuesday, Marloes booked a flight for her next vacation. The ticket cost 150 Euro and she decided to buy an insurance. She thought she might miss the flight with probability 10% and the insurance cost 16 Euro. The insurance will reimburse 150 Euro if she misses the flight. In the evening, she went to Holland Casino and bet 5 Euro on numbers 4, 5 and 6 at a Roulette table. (A roulette gives a number between 0 and 36, with equal probability.) The ball fell on 4 and she received 60 Euro (inclusive the 5 Euro she bet).

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  - Did she use Expected Utility (EU) with a concave utility in both decision? Why?
  - Which model could explain her behavior? How?

The End!

# Today's recommendation

# Today's recommendation



- Fatoumata Diawara's "Fatou" (2011).
- "a wondrous work of cultural preservation from one of the biggest names in contemporary African music" —The economist