Behavioral Economics

Víctor González-Jiménez[1]

[1]Erasmus School of Economics, Erasmus University

Lecture 6: Game Theory

Today's Topics

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1. Exam-like question

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2. Static Games and Nash Equilibrium

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   - Pure Strategies

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5. Empirical Phenomena

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    - Failure to play equilibrium in pure strategies.
        - The traveler's dilemma and coordination games

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     - Matching Pennies

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     - The traveler's dilemma and coordination games
   - Failure to play equilibrium in mixed strategies.
     - Matching Pennies
   - Failure to perform backward induction.
     - Should you trust others to be rational?

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- Values of going to the matches are as in Example 3 of last lecture, and the cost of going to each of the matches is now zero.
- **When do you go to Feyenoord?**

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- Sophistication can worsen self-control problems.

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- **Preproperation**: go for activities with immediate rewards and delayed costs
    - do it when you should wait
        - Smoking
        - Exam-like question

2. Static Games and Nash Equilibrium

Game theory

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- The **payoff function** gives a payoff for each possible combination of strategies

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  - Table describes payoffs of Player 1 and Player 2

		Player 2		
		L	M	R
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- What will a rational player do in this game?

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- Please look at player 2 and compare his strategies L, M and R.

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- Please look at player 2 and compare his strategies L, M and R.
- Is one of the strategies of Player 2 strictly dominated by one of his other strategies?

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  - Player 2: R is dominated by M $\Rightarrow$ Eliminate R.
  - Player 1: D is dominated by U (once R is eliminated) $\Rightarrow$ Eliminate D.

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  - Player 1: D is dominated by U (once R is eliminated) $\Rightarrow$ Eliminate D.
  - Player 2: L is dominated by M (once D is eliminated).
- Player 1 chooses U and Player 2 chooses M. The outcome of the game is (U, M).

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  - Player 1: D is dominated by U (once R is eliminated) $\Rightarrow$ Eliminate D.
  - Player 2: L is dominated by M (once D is eliminated).
- Player 1 chooses U and Player 2 chooses M. The outcome of the game is (U, M).
- this game is **dominance solvable**

A game

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- If they both defect, each will spend six years (-6) in prison.
- **If you were one of the suspects what would you do? Cooperate or defect?** (assume that you only want to minimize the time spent in jail, no risk of revenge)

The prisoner's dilemma

- **Example 2**

	Player 2		
Player 1		C	D
	C	-1,-1	-9,0
	D	0,-9	-6,-6

- strategies are cooperate C or defect D

The prisoner's dilemma

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- strategy C is strictly dominated by D:
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- outcome of the game: (D, D)
- why prisoner's dilemma?
  - (C, C) pareto-dominates by (D, D)

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- strategies are cooperate C or defect D
- strategy C is strictly dominated by D:
  - $-1 < 0$ and $-9 < -6$
- outcome of the game: (D, D)
- why prisoner's dilemma?
  - (C, C) pareto-dominates by (D, D)
- A strategy **pareto-dominates** another if it improves the payoff of at least one player, without hurting the other(s)

The prisoner's dilemma

- **Example 2 (cont'd)**

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Table: Game a)

	Player 2		
Player 1		C	D
	C	9,9	1,10
	D	10,1	4,4

Table: Game b)

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Table: Game a)

	Player 2		
Player 1		C	D
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	D	10,1	4,4

Table: Game b)

- This is the same game (affine transformation)
  - structure is the same
  - magnitude of the numbers not important

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- Example: NE in the Prisoner's dilemma?

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    - No player wants to unilateral deviate from his equilibrium strategy
- Example: NE in the Prisoner's dilemma?
- Movie"A beautiful Mind" about the life of John F. Nash

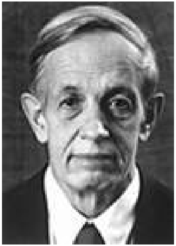

Figure: John F. Nash (Nobel Prize 1994)

How to find a NE?

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- Underline **best responses**

	Player 2		
Player 1		C	D
	C	9,9	1,10
	D	10,1	4,4

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- NE when both payoffs are underlined.

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- NE when both payoffs are underlined.
- Notice that unilateral deviations from the NE are undesirable.

Examples with several NE

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- **Example 3**. Coordination Game

		Player 2	
		A	B
Player 1	A	1,1	0,0
	B	0,0	1,1

Examples with several NE

- **Example 3**. Coordination Game

- Player 2

		A	B
Player 1	A	1,1	0,0
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- Maximum payoffs if both players choose the same
- 2 NEs: (A, A) and (B, B)

Examples with several NE

- **Example 4**. Battle of the Sexes

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		Player 2	
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		Player 2	
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Player 1	B	1,2	0,0
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- Players get higher payoffs if they choose the same but they have different tastes (as, sometimes, in a couple. . .)
- 2 NEs: (B, B) and (S, S)

Examples with several NE

- **Example 5**. Pareto-dominance vs. risk-dominance

Examples with several NE

- **Example 5**. Pareto-dominance vs. risk-dominance

		Player 2	
		L	R
Player 1	U	9,9	0,8
	D	8,0	7,7

Examples with several NE

- **Example 5**. Pareto-dominance vs. risk-dominance

		Player 2	
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Player 1	U	9,9	0,8
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- 2 NEs: (U, L), (D, R)
- (U, L) pareto-dominates (D, R)

Examples with several NE

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		Player 2	
		L	R
Player 1	U	9,9	0,8
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- 2 NEs: (U, L), (D, R)
- (U, L) pareto-dominates (D, R)
- But it is riskier
  - If P2 goes for R, P1 would end up with 0!
  - If P1 goes for D, P2 would end up with 0!

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- But it is riskier
  - If P2 goes for R, P1 would end up with 0!
  - If P1 goes for D, P2 would end up with 0!
- NE does not tell us which one to play!

Mixed Strategies

Mixed Strategies

- **Example 6**. Matching Pennies
  - P2: wins if match
  - P1: wins if do not match

Player 2

		H	T
Player 1	H	-1,1	1,-1
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Mixed Strategies

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- Written as
  - ((prob. P1 plays H, prob. P1 plays T),(prob. P2 plays H, prob. P2 plays T))
- NE in mixed strategies $((1/2, 1/2), (1/2, 1/2)) \Rightarrow$ **Be unpredictable!**

3. Dynamic Games

Extensive form versus strategic form

- Strategic form (normal form)
  - The players choose simultaneously
  - Players choose without knowing what the other player has done (before)

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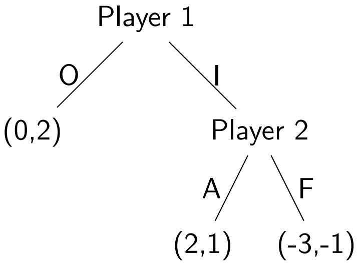
- Strategic form (normal form)
  - The players choose simultaneously
  - Players choose without knowing what the other player has done (before)
- In some situations players observe other players moves before they move – **extensive form**

Subgame perfection: an example



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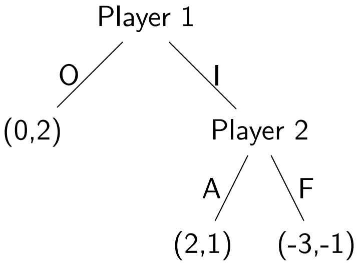
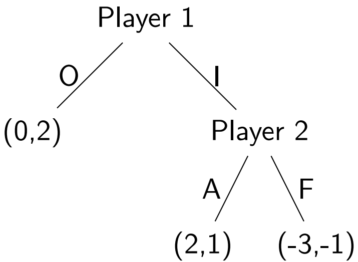


- **Example 7.**
  - Extensive form: game tree
  - A **subgame** of a game is any part of that game which in itself constitutes a game.
    - Subgames: 2

Subgame perfection: an example

- **Example 7**
  - Find NE (use normal form)



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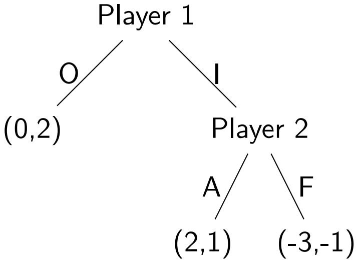

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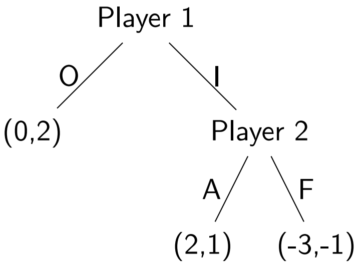

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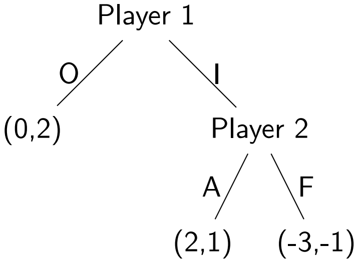

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- 2 NE: (O, F) and (I, A).
- one of these equilibria contains a *not credible* threat.
- this is not rational.

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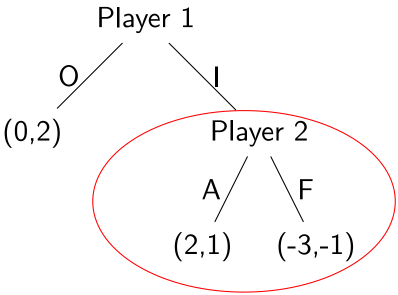
- Subgame-perfect equilibria can be found using **backward induction**.
- Idea: Eliminate equilibria with threats that are **not "credible"**.
    - if a strategy profile is not an equilibrium later on in the game (i.e., in a subgame) then it is not credible

Subgame perfection: an example

- **Example 7**



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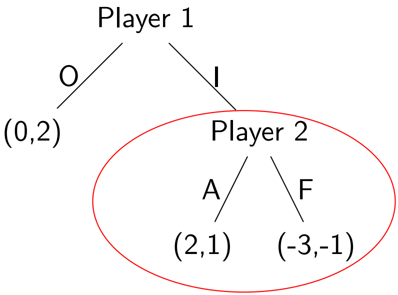

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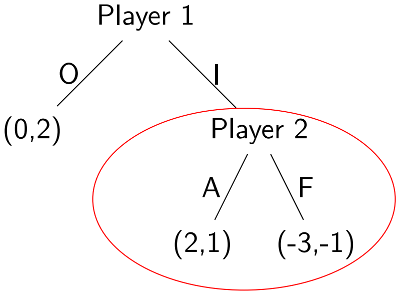

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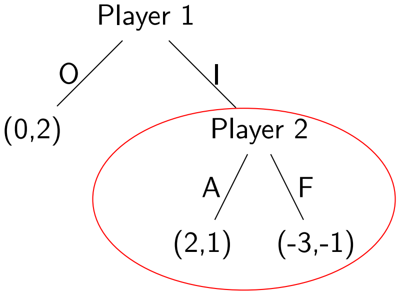

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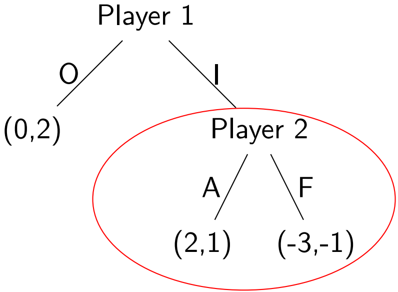

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- subgame-perfect NE: (I, A)

4. Assumptions in Game Theory

Rationality

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- People are motivated by pure self-interest
  - All players choose the strategy by maximizing their own utility
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- Formal definition by Robert Aumann (1976).

5. Empirical Phenomena.

- Failure to play equilibrium in pure strategies.
  - The traveler's dilemma and coordination games.
- Failure to play equiibrium in mixed strategies.
  - Matching Pennies
- Failure to perform backward induction.
  - Should you trust others to be rational?

Results taken from

- Goeree, Jacob K. Charles Holt (2001), "Ten Little Treasures of Game Theory and Ten Intuitive Contradictions," The American Economic Review 91 no. 5, 1402–1422.
- Your own decisions :)

The traveler's dilemma

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- Two players simultaneously choose integer numbers from the interval $[180, 300]$
- Payoff: the players are paid the **lower of the two** numbers **and** an amount of $R > 1$ is transferred from the player with the higher number to the player with the lower number

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    - 300 is never a best response (if the other says 300, I can get $\geqslant 300$ by saying 299)

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  - But then, if the other says 299, I can get more by saying 298. So 299 ruled out as well!

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  - But then, if the other says 299, I can get more by saying 298. So 299 ruled out as well!
  - ...
  - Lower bound 180 survives this process and is the unique NE

Experimental results with 2 different R

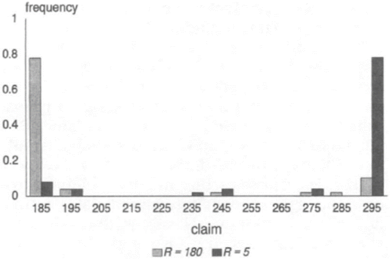

FIGURE 1. CLAIM FREQUENCIES IN A TRAVELER'S DILEMMA FOR R = 180 (LIGHT BARS) AND R = 5 (DARK BARS)

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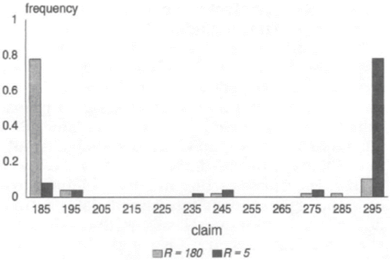

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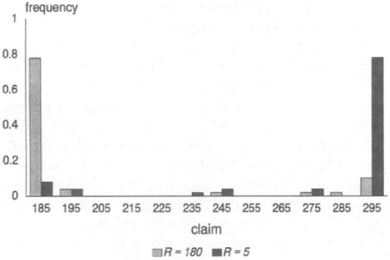

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  - Average choice 280
  - Around 80% choose the highest possible claim of 300

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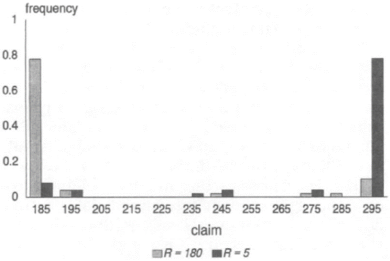

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- **What do you think happened?**
- 76% of you chose option A, 52% groups coordinated on either on (A, A) or (B,B).

Coordination games and focal points

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- Back to **Example 3**

		Player 2	
		A	B
Player 1	A	1,1	0,0
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- In this case: 2 NE
  - In general: many NE

Coordination games and focal points

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    - Some equilibria tend to stand out in the minds of players.
    - E.g. option A!

A slightly modified coordination game

- Consider two people. They can either choose A or B. If their choices coincide, they both receive a positive payoff (if both choose A, both receive 100, if both choose B, both receive 200), if the two choices do not coincide, both receive 0.

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- **What do you think happened?**
- 82% of you chose option B, 66% groups coordinated on either on (B, B).

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		Player 2	
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- (A, A) is pareto-dominated by (B, B)
  - Pareto optimal equilibrium (B, B) is **focal**.

Another coordination game

- You are to meet somebody in Rotterdam. You have not been instructed where to meet; you have no prior understanding with the person on where to meet; and you cannot communicate with each other. You are simply told that you will have to guess where to meet and that he is told the same thing and that you will just have to try to make your guesses coincide.

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- Where would you go? When would you go?

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- **Focal points:**
  - 15 (48%) of you wrote Rotterdam Centraal
  - 6 (22%) of you wrote Market Hall
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Another coordination game

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  - 1 of you wrote " my house the whole day" :/

Do we play equilibria in mixed strategies?

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- As Player 1, what strategy would you choose?

		Player 2 L	Player 2 R
Player 1	T	320,40	40,80
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- **79% of you chose "Top"**

Matching Pennies

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Table: Treasure treatment

		Player 2 L	R
Player 1	T	80,40	40,80
	B	40,80	80,40

- Symmetric matching pennies game (similar to that of **Example 6**).
  - NE in mixed strategies $((1/2, 1/2), (1/2, 1/2))$
- **Treasure treatment**
  - Goeree and Holt (2001) find choice percentages 50-50

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- **Intuition**: row players are attracted by 320. . . , become predictable and are exploited!

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  - Take the sequence $T - H - H - T - H - T - T - H - T - H$
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- Paul W. Glimcher (2003), "Decisions, Uncertainty and the Brain: The Science of Neuroeconomics", Cambridge, MA: MIT Press.

Who would you choose to take a penalty if your life depended on it?

a) Maradona
b) Myself
c) This would never happen, so the question is ridiculous

Proper Randomization

Proper Randomization

- Experts
- **Football** - direction of the penalty kick (both: player and goalkeaper)
  - Palacios-Huerta, Ignacio (2003), "Professionals play minimax", The Review of Economic Studies 70, 2, 395-415.
    - finds that Zidane kicks penalties to L with 0.48 probability and to R with 0.52 probability.
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- **Main findings: successful randomization**

Proper Randomization

- Palacios-Huerta, Ignacio, Maradona Plays Minimax (May 7, 2021). Available at: https://ssrn.com/abstract=3841354
  - finds that Maradona kicks penalties to L with 55% chance and R with 45% chance.
  - Keepers facing Maradona choose L with 45% chance and R with 55% chance.

Should you trust others to be rational?

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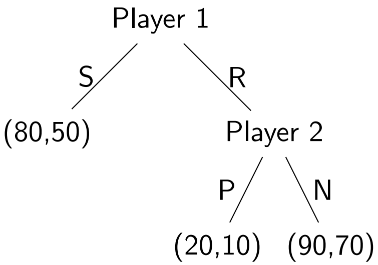

- Consider this dynamic game

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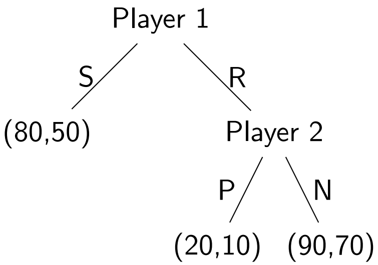

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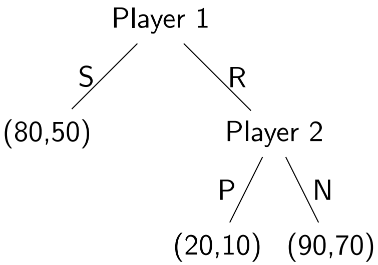

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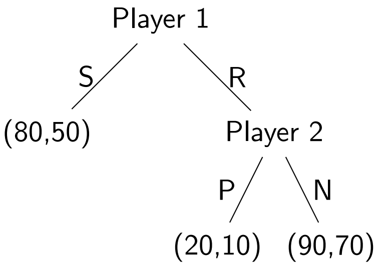

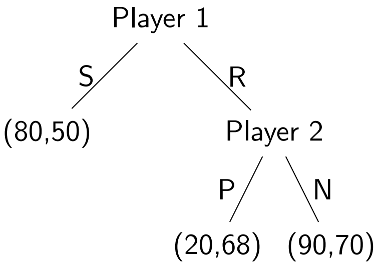

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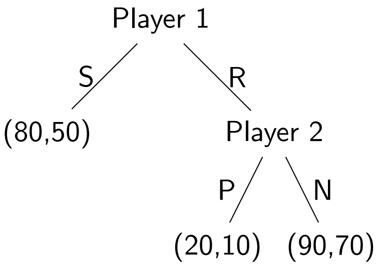

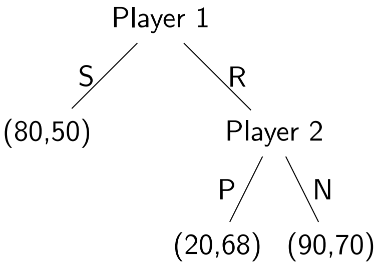

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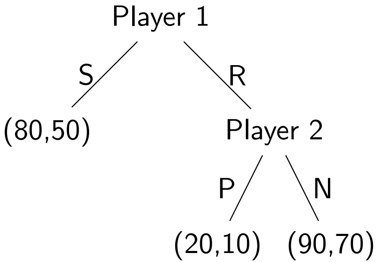

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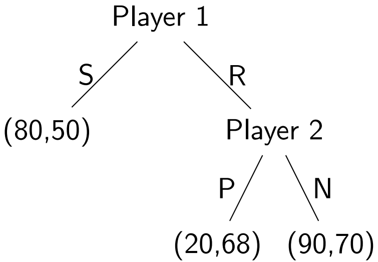

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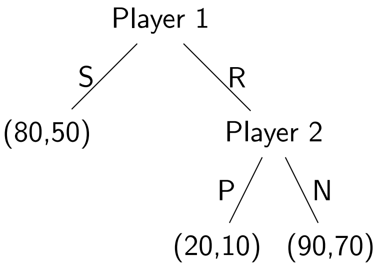

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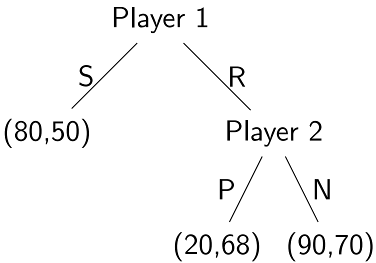

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- In the previous game, Player 2 was better off if Player 1 let him move (by choosing R)
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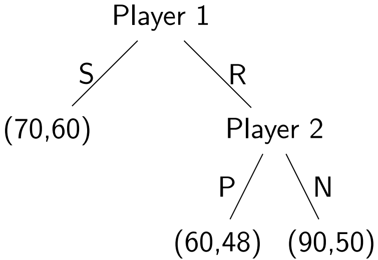

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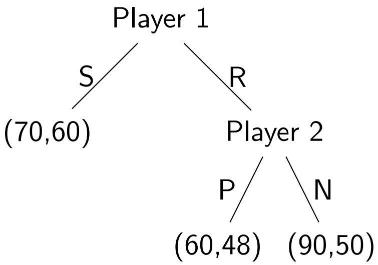

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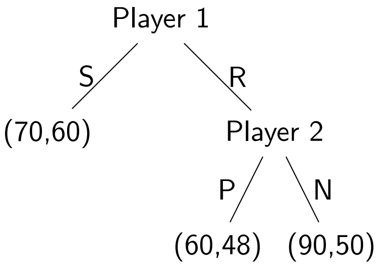

- **As Player 1, what would you choose?**
  - 77% of you chose S.

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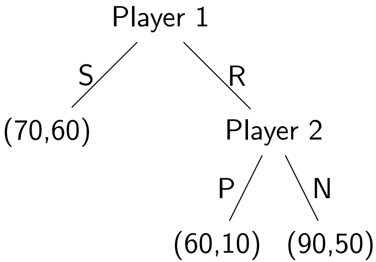

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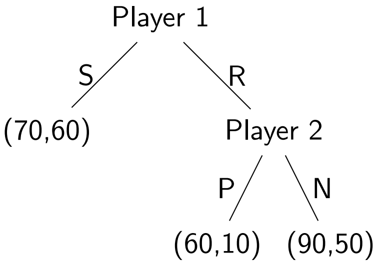

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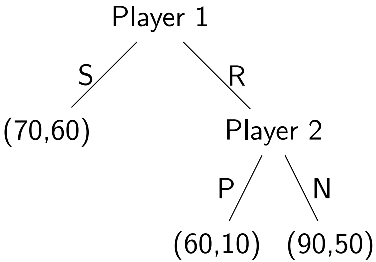

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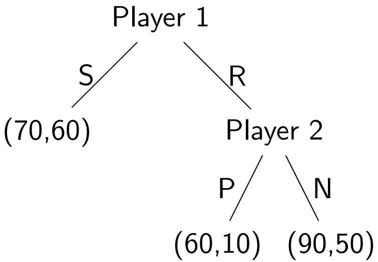

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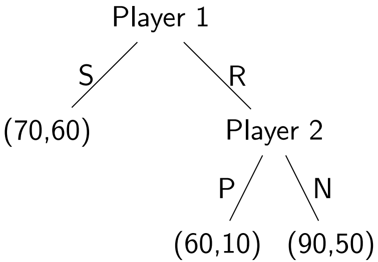

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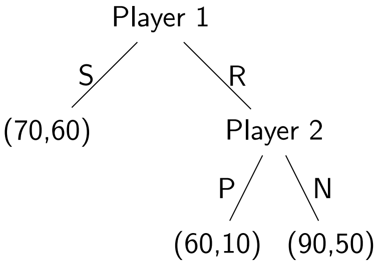

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  - **The threat is not credible**

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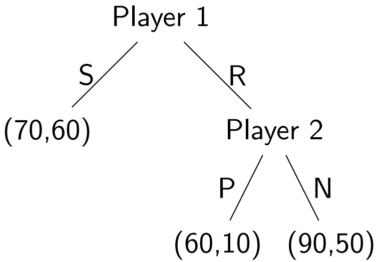

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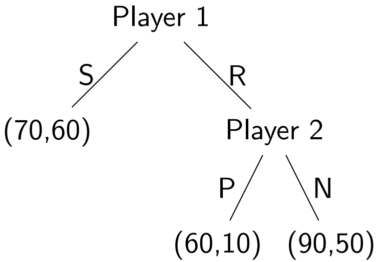

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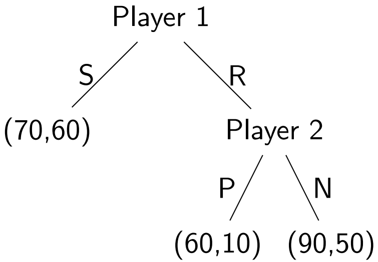

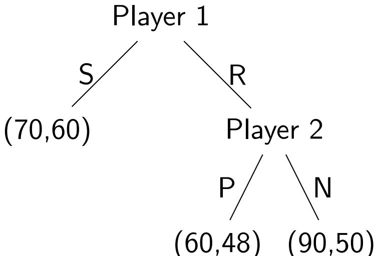

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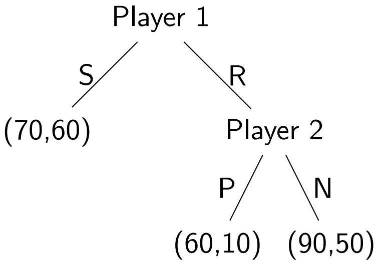

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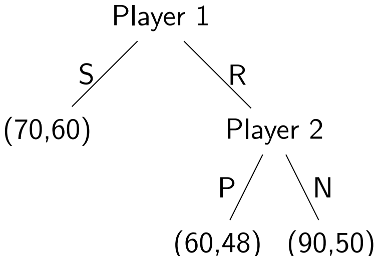

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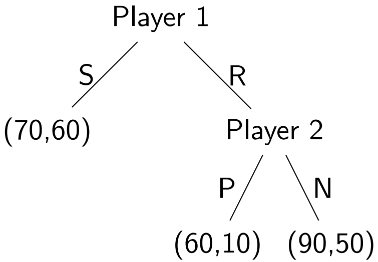

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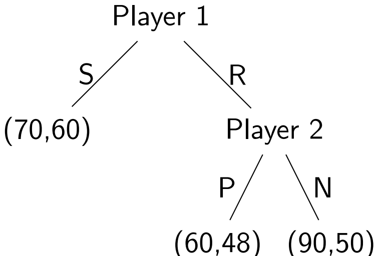

- **Contradiction treatment** (cheap punishment: 2)
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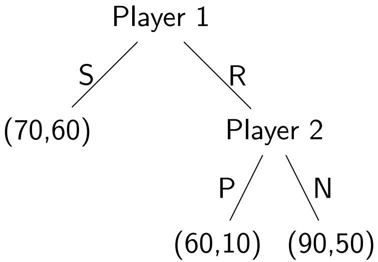

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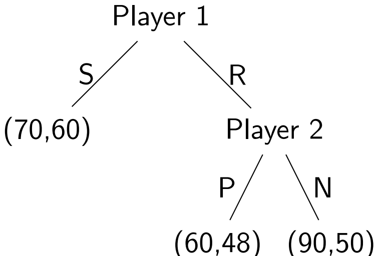
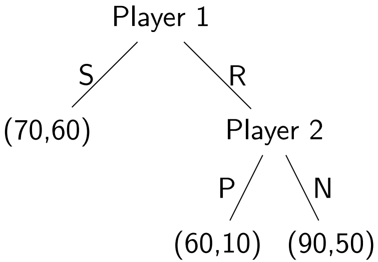
  - In Goeree and Holt (2001) 88% P1s choose R.
- No, you should not believe a non-credible threat.



- **Contradiction treatment** (cheap punishment: 2)
- still non-credible threat
  - 68% P1 choose R.

Should you believe in a threat that is not credible?



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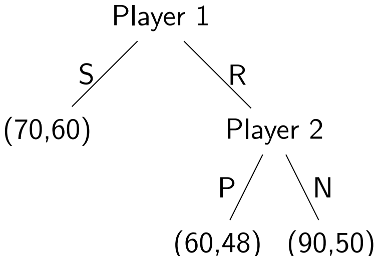

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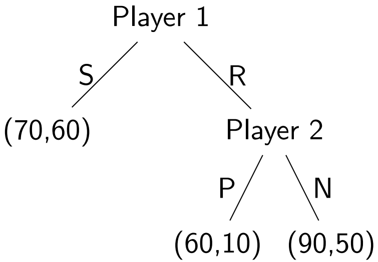

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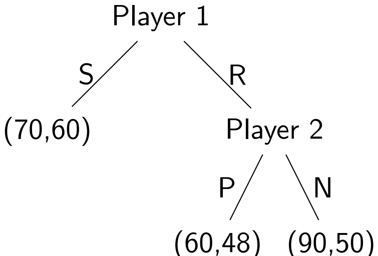

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- **Yes, you should believe...?**

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Exam-like exercise

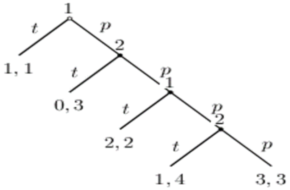

Figure: Game

- The game:
  - Two players, $1, 2$, choose alternately

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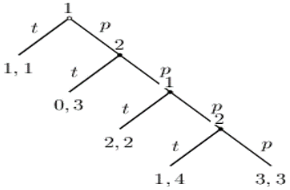

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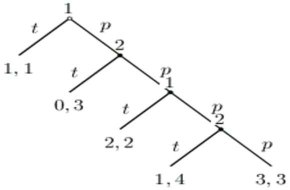


Figure: Game

- The game:
  - Two players, $1, 2$, choose alternately
  - At each stage, a player can
    - choose t and end the game
    - or choose p and move on
  - The game has 4 stages

Exam-like exercise

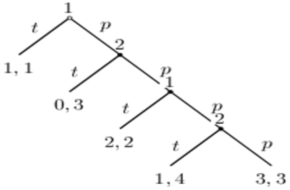
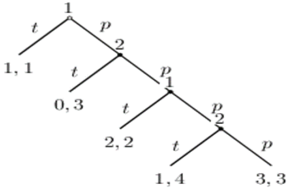


Figure: Game

a Find the game theoretic solution of the game: Use backward induction to find the unique subgame perfect equilibrium.

Exam-like exercise



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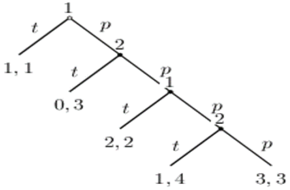

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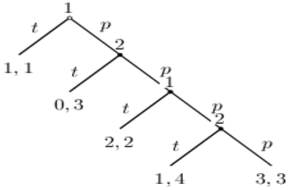

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d Can you think of explanations of the behavior? Give an intuition.

The End!

Today's recommendation

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- Thrice's "Horizons/East" (2021).
- **Interesting economics fact:** Each album released by Thrice has had a portion of its sales proceeds donated to a new charitable organization (not self-interested at all!)