Behavioral Economics

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Lecture 8: Behavioral Game Theory

Today's Topics

1. Exam-like exercise.

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2. Small introduction.

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4. Behavioral Model: Level-k.
    - Explains behavior in:
        - Beauty Contest.
        - Battle of the Sexes and Coordination Games.

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   - Explains behavior in:
     - Beauty Contest.
     - Battle of the Sexes and Coordination Games.
5. Behavioral Model: Quantal Response Equilibrium.
   - Explains Behavior in:
     - Matching Pennies.
     - Centipede Game.

Exam-like exercise: Dictator Game with Punishment

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- The Game

Exam-like exercise: Dictator Game with Punishment

- The Game
  - Dictator Ann divides 10 between herself and Bill. She keeps y.
  - Bill receives the remainder $10 - y$
  - Punisher Charles observes the dictator game and can punish Ann by m Euro. It is costless to him. He receives 5 anyhow.

Exam-like exercise: Dictator Game with Punishment

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  - A's payoffs: $X_A = y - m$.
  - B's payoffs: $X_B = 10 - y$.
  - C's payoffs: $X_C = 5$.

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  - B's payoffs: $X_B = 10 - y$.
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- Charles Utility

$$u_C(X_A, X_B, X_C) = X_C - \alpha \cdot \big(\max(X_A - X_C, 0) + \max(X_B - X_C, 0) \big) - \beta \cdot \big(\max(X_C - X_A, 0) + \max(X_C - X_B, 0) \big)$$

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   - So $6 – m = 5$ and punishment is set to $m = 1$

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b How much will Charles punish if Ann proposes to give 2 Euro to Bill?

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Exam-like exercise: Dictator Game with Punishment

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- Same reasoning as in a).
- $8 – m = 5$ and punishment is set to $m = 3$.

2. Small Introduction

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B Being always able to have correct beliefs.

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- We will study two models that relax these assumptions.
- After this lecture you can determine if your choice of superpower was satisfactory.
  - By evaluating whether the model you consider the least "realistic" does not relax your superpower.

3. The Beauty Contest Game

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  - ...
  - iterating this rationale, the NE is 0.

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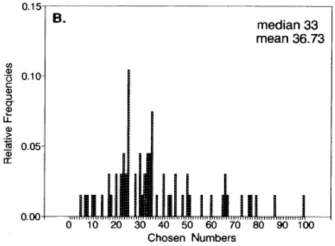
- Treatment assignment in this experiment was at the session level.
    - $2/3$ **of the average (Sessions 1-3).**
        - We focus on this game.
    - $1/2$ of the average (Sessions 4-6).
    - $3/4$ of the average. (Sessions 7-9).
- Prize for the winner was $\$13$.

Results of the Beauty Contest Game



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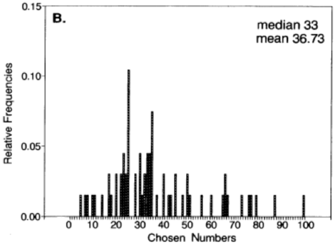

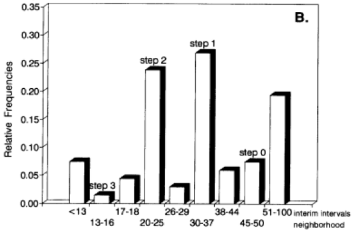

Figure: Results the Beauty Contest

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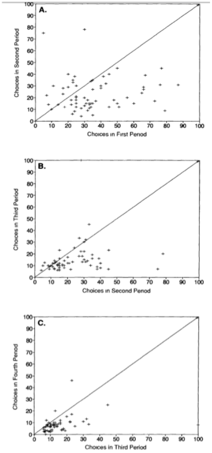

FIGURE 4. OBSERVATIONS OVER TIME FOR SESSIONS 4–7 ($p = 2/3$): A) TRANSITION FROM FIRST TO SECOND PERIOD; B) TRANSITION FROM SECOND TO THIRD PERIOD; C) TRANSITION FROM THIRD TO FOURTH PERIOD

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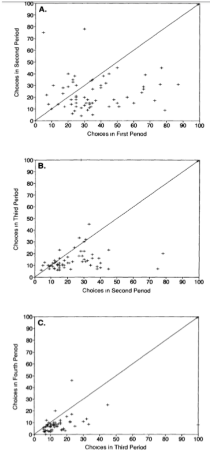

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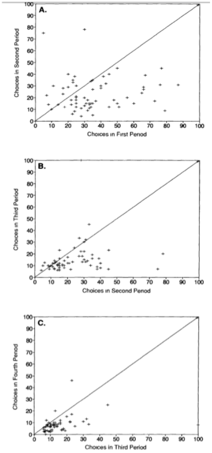

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- Behavior approaches equilibrium in latter periods.
- Suggests that responses are done in finite steps rather than infinitely iterating .

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  - in this model beliefs and choices of players need not to be consistent.
  - recall that in the NE beliefs are consistent.
    - I play the best response to what I believe is the best response of the other players.
- The rationale behind level-k: *"why are the actions of other players, who I never met before, need to be consistent with my beliefs?"*

4. Behavioral Model: Level-k Model

- Explains behavior in:
  - Beauty Contest.
  - Battle of the Sexes.

Predictions of the level-k model in the Beauty Contest Game

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  - Consider the version of the game in which $p = \frac{2}{3}$ and $x_i \in [0, 100]$.

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  - Thus the average guess of players L_0 is $\bar{x}_{L0} = 50$.

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- The heterogeneity of players can explain the behavior observed in Beauty Contest.
  - There, most people are L_1 and L_2.
  - **Including you ;)**

Beauty Contest Game with other Subjects

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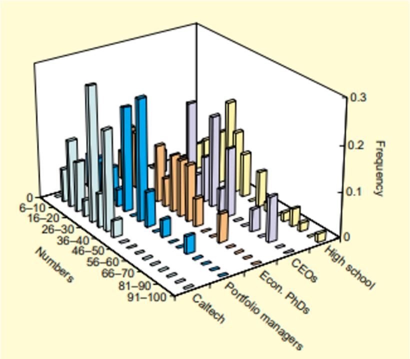

- Similar "spikes" as with " regular" students.

Cognitive ability and Level-k

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- to measure cognitive ability they use a Raven's matrices test
  - used to measure intelligence.
- then they let players play the beauty contest game.

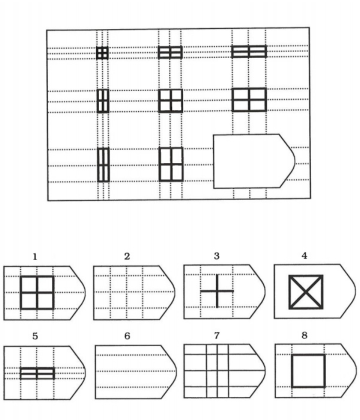
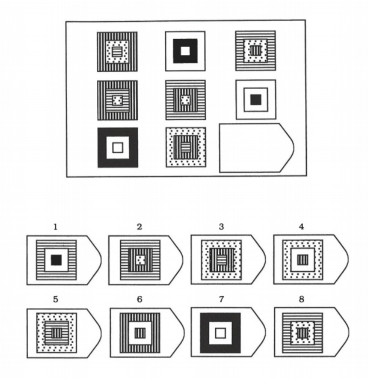


Figure: Example of a Raven's matrix



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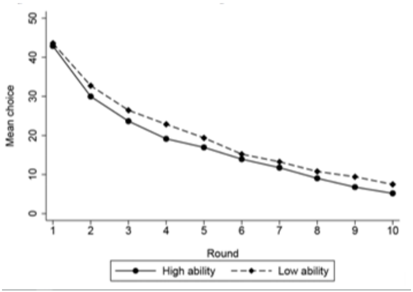
Cognitive ability and Level-k



- More cognitively able subjects choose numberscloser to equilibrium, earn more, and converge more frequently toequilibrium play

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  - However, coordination was not too high ($\approx 50\%$).
    - The power of focal points is limited.

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- Consider the following "battle of the sexes" game with $a > 1$

	Player 2	
Player 1		H	D
	H	(0,0)	(a,1)
	D	(1,a)	(0,0)

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- Coordination when outcomes of (H, D) or (D, H) are obtained.

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  - They choose H
- Types L_4 best respond conditional on the strategy of L_3.
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Predictions of the level-k model in the Battle of the Sexes

- The interaction between types results in different equilibria as shown in the following table.

Table: Equilibrium actions in a level-K model without communication

Types	L1	L2	L3	L4
L1	H, H	H, D	H, H	H, D
L2	D, H	D, D	D, H	D, D
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- If a type L_3 plays against a L_2 player then the resulting outcome is (H, D).
- Coordination is only possible if L_1 or L_3 play against L_2 or L_4.
- Cooperation depends on the frequency of types.

Predictions of the level-k model in Coordination games

- Cooperation is entirely accidental in the level-k model.
  - This could explain why the coordination rate in the game of Lecture 6 was close to 50%.

Coordination games

- In lecture 6, we also discussed thefollowing game:

		Player 2 A	B
Player 1	A	1,1	0,0
	B	0,0	2,2

- most of you chose B.
  - we concluded that B becomes focal.

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  - L_0 players randomize between A and B, and Players L_1 best respond by choosing B.

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  - we concluded that B becomes focal.
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- outcome (B, B) is profitable for all types.

Problems with level-k model

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Problems with level-k model

- What is the distribution of types?
- Are types fixed?
- Why do players think others are one level below? Why not two levels below?

NE in Mixed Strategies (Review)

Matching Pennies Game

Matching Pennies Game

- Recall **Example 6** from **Lecture 6**:

	Player 2		
Player 1		L	R
	T	(-1,1)	(1,-1)
	B	(1,-1)	(-1,1)

Matching Pennies Game

- Recall **Example 6** from **Lecture 6**:

	Player 2		
Player 1		L	R
	T	(-1,1)	(1,-1)
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- No NE in pure strategies, but one equilibrium in mixed strategies.

Matching Pennies Game

- Denote by π_R the probability that Player 2 plays R, and π_T the probability Player 1 plays T.

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Matching Pennies Game

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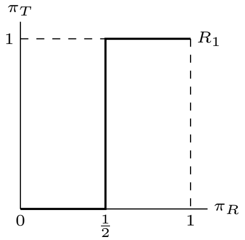

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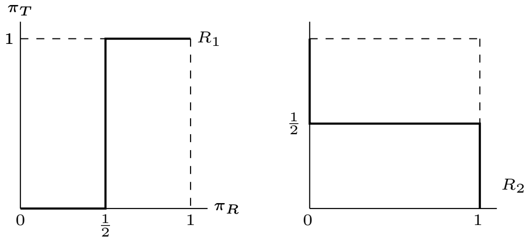

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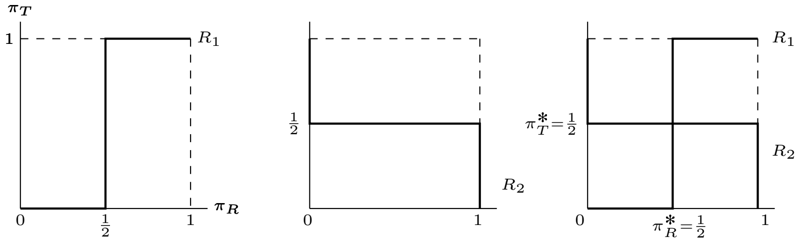

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	Player 2		
Player 1		L	R
	T	(-1,1)	(3,-1)
	B	(1,-1)	(-1,1)

- What are the Nash Equilibria of this game?

Mixed strategies

- Player 1 prefers T to B if $\pi_R \geqslant \frac{1}{3}$

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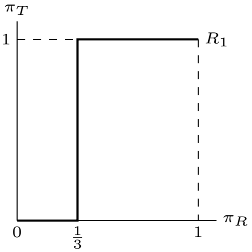

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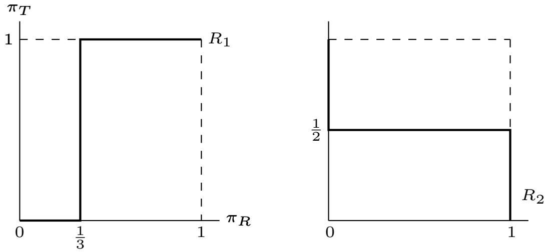

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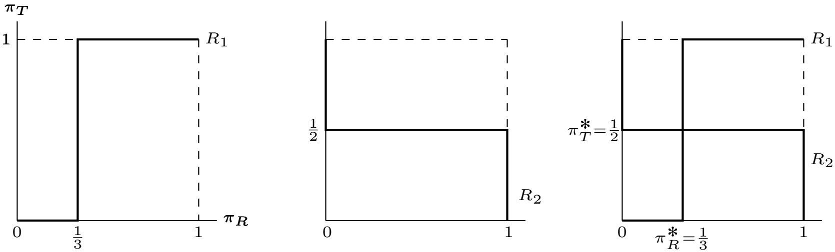

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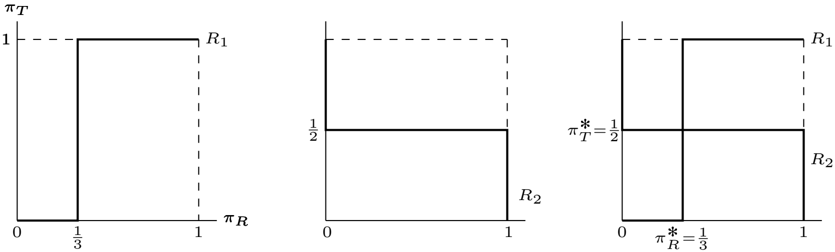

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Do people play mixed strategies?

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- Recall the following game from **Lecture 6**:

	Player 2	
Player 1	L	R
T	(80,40)	(40,80)
B	(40,80)	(80,40)

Do people play mixed strategies?

	Player 2		
Player 1		L **(48%)**	R **(52%)**
	T **(48%)**	(80,40)	(40,80)
	B **(52%)**	(40,80)	(80,40)

- Close to 50% of subjects play each of the strategies.

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	Player 2		
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- Close to 50% of subjects play each of the strategies.
- This is in line with the nash equilibrium in mixed strategies, which is $(1/2, 1/2)$.

Do people play mixed strategies?

- Subjects played the following modified version of the game.

	Player 2		
Player 1		L	R
	T	(320,40)	(40,80)
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	Player 2		
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- The nash equilibrium in mixed strategies of the above game is $(\frac{1}{2}, \frac{1}{8})$.
- However, subjects chose T more often than what the NE predicts.

	Player 2		
Player 1		L **(16%)**	R **(84%)**
	T **(96%)**	(320,40)	(40,80)
	B **(4%)**	(40,80)	(80,40)

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- Research has shown that this could be due to different causes
  - Individuals inability to generate random sequences.
  - Humans' behavior is affected by framing effects.
  - **Players tend to increase the probability of playing an action which gives them a higher payoff**
    - We now focus on this regularity.

Behavioral Model

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  - In other words, very costly errors are unlikely.
- Beliefs are still consistent: agents are aware of the fact that others make mistakes when they compute expected payoffs (unlike the level-k model).

5. Behavioral Model: Quantal Response Equilibrium.

- Explains behavior in:
  - Matching Pennies
  - Centipede Game

Quantal Response Equilibrium

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- **Setting:**
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  - $u_{ik}(\pi_{-i})$ is the *expected payoff* of player i when playing strategy s_k and others play the mixed strategy π_{-i}.
- A strategy s_k is, from the viewpoint of the agent, evaluated by $u_{ik}(\pi_{-i}) + \varepsilon_{ik}$
  - Player i is better off playing k if $u_{ik}(\pi_{-i}) + \varepsilon_{ik} \geqslant u_{ij}(\pi_{-i}) + \varepsilon_{ij}$ for all $j = 1, ..., m$ pure strategies.

The role of ε_{ik}

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- Factors only observable to the agent himself
    - Distractions;

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  - Heterogeneous preferences (spite, envy, altruism).
- The distribution of ε is common knowledge (known by all players).

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  - $\{\sigma_{i1}(\pi_{-j}), ..., \sigma_{im}(\pi_{-j})\}$ is a quantal response function.
- In a quantal response equilibrium (QRE):
  - each individual plays her noisy best response, taking full account for errors made by opponents.

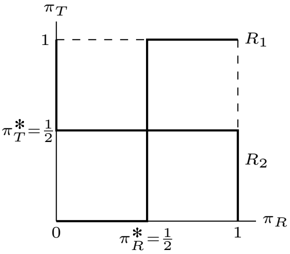
Quantal Response Equilibrium

- An important property of QRE is that σ_{ik} increases in u_{ik}.
  - Strategies with higher payoffs are chosen more often than those with lower payoffs

Matching Pennies and QRE

Matching Pennies and QRE

- Consider the standard matching pennies game.
- Let us draw the Quantal Response functions.



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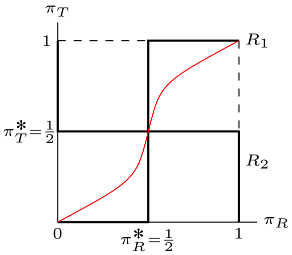

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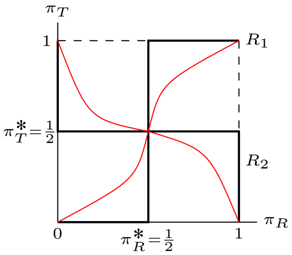

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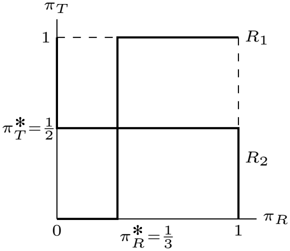

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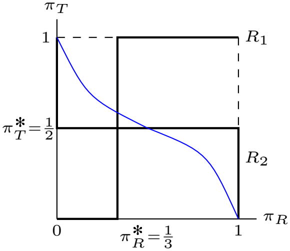

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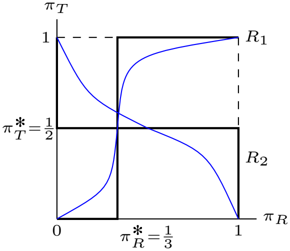

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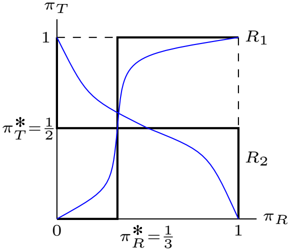

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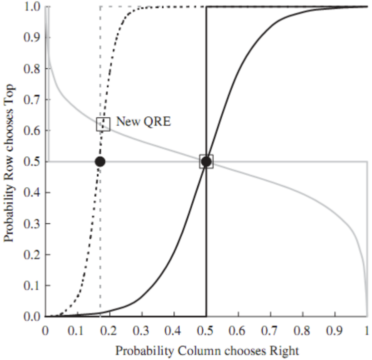

- The NE and the QRE do not longer coincide.
- The QRE is yields a higher probability of Player 1 playing T.
  - The model can explain the tendency of subjects to choose T more often.

What do we assume about ε_{ik}?

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- Typically, one assumes that the distribution of ε_{ik} is $\exp(-\exp(-\lambda\varepsilon_{ik}))$
    - extreme value type 1 distribution.
- Thus, the probability of playing strategy k is $\sigma_{ik} = \frac{\exp(\lambda u_{ik})}{\sum_{l \in A} \exp(\lambda u_{il})}$.

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Table 1
Payoff tables for Games A–D

	Game A		Game B		Game C		Game D	
	L	R	L	R	L	R	L	R
U	9,0	0,1	9,0	0,4	36,0	0,4	4,0	0,1
D	0,1	1,0	0,4	1,0	0,4	4,0	0,1	1,0

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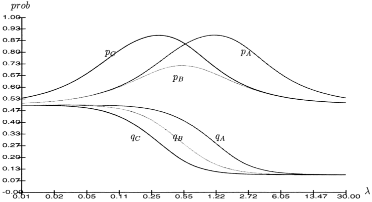
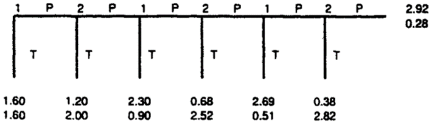


Fig. 1. QRE correspondence for Games A, B, and C.

Centipede game and QRE

Centipede game and QRE

- Rembember the game discussed at the beginning of Lecture 7.



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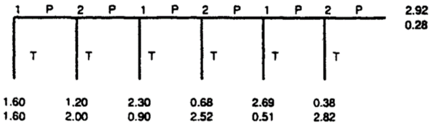

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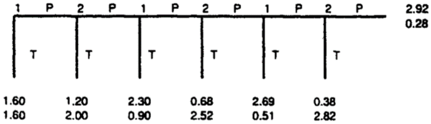

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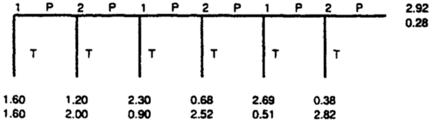
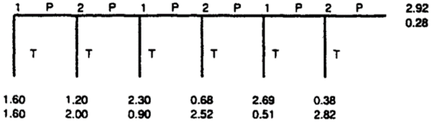
The centipede game



- In the original experiment only 7% of subjects played T in the first node.

The centipede game



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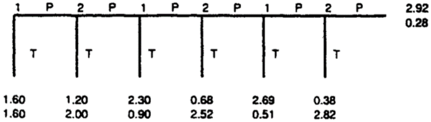

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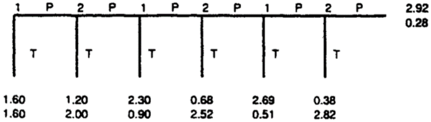

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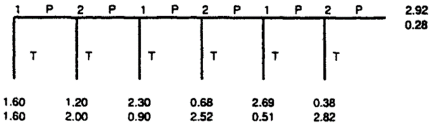

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Explaining behavior in the Centipede game with QRE

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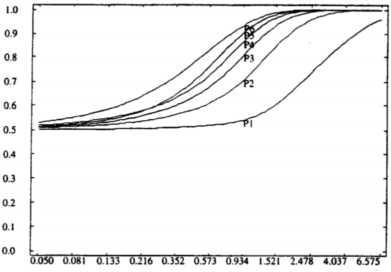

Fig. 5. Quantal response equilibrium of the six-move constant-sum centipede game

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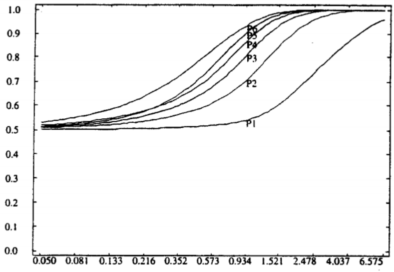

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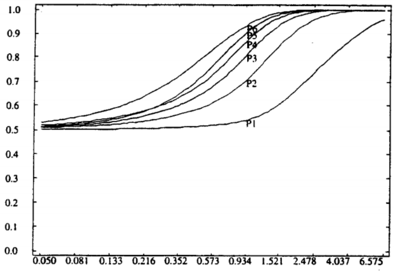

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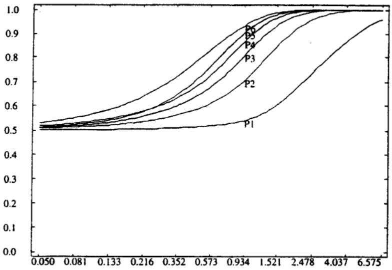

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The End!